Non-Ideal Behavior of Components

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Circuit Schematics
Resistors
Resistance

\[ R = \frac{l}{\sigma A} \text{ ohms} \]
Quiz Question

The D.C. resistance of a 5-cm trace on a printed circuit board is,

a.) about 100 ohms

b.) about 100 milliohms

c.) less than a milliohm
DC Resistance of a Printed Circuit Board Trace

Trace length = 5 cm
Trace width = 0.25 mm
Trace thickness = 0.034 mm
$\sigma_{cu} = 5.7 \times 10^7$ S/m.

\[
R = \frac{0.05m}{(5.7 \times 10^7 \text{ S/m})(0.25 \times 10^{-3} \text{ m})(0.034 \times 10^{-3} \text{ m})} = 0.10 \text{ ohms}
\]
Skin Depth

High-frequency electric fields and currents decay exponentially with distance from the surface of a good conductor.

\[ J(x) = J_s e^{-x\sqrt{\pi f \mu \sigma}} \]
Skin Depth

\[ \delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \text{ meters} \]

Current density in within wire approximated as a uniform current penetrating one skin depth into the surface.
Resistance per unit length of a cable

At 60 Hz

\[ \delta_{60Hz} = \frac{1}{\sqrt{\pi(60)(4\pi \times 10^{-7})(5.7 \times 10^7)}} = 8.6 \text{ mm} \]

\[ R_{\text{inner conductor at 60Hz}} = \frac{1}{\sigma A} = \frac{1}{(5.7 \times 10^7)\pi(0.5 \times 10^{-3})^2} = 22.3 \text{ m}\Omega / \text{m} \]

\[ R_{\text{outer conductor at 60Hz}} = \frac{1}{\sigma A} = \frac{1}{(5.7 \times 10^7)2\pi(5 \times 10^{-3})(0.1 \times 10^{-3})} = 5.6 \text{ m}\Omega / \text{m} \]

\[ \text{total} = 22.3 + 5.6 = 28 \text{ m}\Omega / \text{m} \]
Resistance per unit length of a cable

At 100 MHz

\[
\delta_{100\,\text{MHz}} = \frac{1}{\sqrt{\pi \left(10^8 \right) \left(4\pi \times 10^{-7} \right) \left(5.7 \times 10^7 \right)}} = 0.0066 \text{ mm}
\]

\[
R_{\text{inner conductor at 100 MHz}} = \frac{1}{\sigma A} = \frac{1}{\left(5.7 \times 10^7 \right) 2\pi \left(0.5 \times 10^{-3} \right) \left(6.6 \times 10^{-6} \right)} = 832 \, \text{m}\Omega / m
\]

\[
R_{\text{outer conductor at 100 MHz}} = \frac{1}{\sigma A} = \frac{1}{\left(5.7 \times 10^7 \right) 2\pi \left(4.9 \times 10^{-3} \right) \left(6.6 \times 10^{-6} \right)} = 85 \, \text{m}\Omega / m
\]

\[
\text{total} = 832 + 85 = 917 \, \text{m}\Omega / m
\]
Capacitors
**Capacitance**

\[ E = \frac{Q_0}{4\pi \varepsilon_0 r^2} \quad r_a < r < r_b \quad \text{volts/m.} \]

\[ V_{ab} = \int_{r_a}^{r_b} \frac{Q_0}{4\pi \varepsilon_0 r^2} \, dr = \frac{Q_0}{4\pi \varepsilon_0} \left[ \frac{1}{r_a} - \frac{1}{r_b} \right] \quad \text{volts.} \]

\[ C_{ab} = \frac{Q_0}{V_{ab}} = \frac{4\pi \varepsilon_0}{\left[ \frac{1}{r_a} - \frac{1}{r_b} \right]} \quad \text{farads.} \]

\[ r_a = 0.5 \text{ mm} \]
\[ r_b = 4.9 \text{ mm} \]
Absolute Capacitance

\[ C_{abs} = \lim_{{b \to \infty}} C_{ab} = 4\pi \varepsilon_0 r_a \text{ farads.} \]

\( r_a = 0.5 \text{ mm} \)
\( r_b = 4.9 \text{ mm} \)
Self and Mutual Capacitance

\[ C_{11}, C_{12}, C_{13}, C_{22}, C_{23}, C_{33} \]
Self and Mutual Capacitance
Inductors
Inductance

Inductance is a property of current loops!

\[ \Psi = \int B \cdot ds \quad \text{webers} \]

\[ L = \frac{\Psi}{I} \quad \text{henries} \]
Inductance

\[ L_{\text{circle}} \approx N^2 R \mu \left[ \ln \left( \frac{8R}{a} \right) - 2.0 \right] \text{ henrys} \]

\[ L_{\text{square loop}} \approx N^2 \frac{2\mu_0 w}{\pi} \left[ \ln \left( \frac{w}{a} \right) - 0.774 \right] \text{ henrys} \]
Quiz Question

The inductance of a 2-cm wide, 10-cm long ground strap is,

a.) about 10 nanohenries
b.) about 100 nanohenries
c.) undefined
Loop Inductances

Length of side = 7.5π cm
wire radius = 1 mm

Loop radius = 3 cm
wire radius = 1 mm

Length of side = 1.5π cm
wire radius = 1 mm

Loop radius = 3 mm
wire radius = 0.1 mm

L = 130 nH
6.2 nH/cm

L = 116 nH
6.2 nH/cm

L = 13 nH
7.0 nH/cm

L = 580 nH
6.2 nH/cm
Mutual Inductance

\[ L_{21} = \frac{\Psi_{21}}{I_1} \text{ henries.} \]
**Partial Inductance**

\[ L_{2l} = \frac{\Psi_{2l}}{I_l} \text{ henries.} \]

\[ L_{2l} = \frac{\int B_1 \cdot ds}{I_l} \text{ henries.} \]

\[ L_{2l} = \frac{\int (\nabla \times A_1) \cdot ds}{I_l} = \frac{\int A_1 \cdot dl_2}{I_l} \text{ henries} \]

\[ A_i = \oint \frac{\mu I_1}{4\pi R} dl \text{ webers/m} \]

\[ L_{2l} = \frac{\mu}{4\pi} \oint \oint \frac{dl_1 \cdot dl_2}{R} \text{ henries} \]

\[ L_{ij} = \frac{\mu}{4\pi} \sum_{i=0}^{I} \sum_{j=0}^{J} l_{ij} \]

**Useful for computer modeling.**

**Not useful for estimating inductance.**

where \[ l_{ij} = \int_{\text{segment i}} \int_{\text{segment j}} \frac{dl_i \cdot dl_j}{R_{ij}} \]
Partial Inductance (Branch Inductance)

\[ L_{\text{loop}} = L_{\text{trace}} + L_{\text{via}} + L_{\text{via}} + L_{\text{plane}} \]
Resistors
Do Resistors Have Capacitance and Inductance?
Impedance of a 50-Ohm Resistor

Diagram showing a circuit with a 48 ohm resistor, 0.8 pF capacitor, and 20 nH inductor. A graph plots impedance in Ohms against frequency in MHz.
Types of Resistors

- **Metal Film**
  
  High precision, low cost

- **Composite**
  
  Medium precision, good transient immunity

- **Wire Wound**
  
  High power, high inductance
Capacitors
Do Capacitors Have Resistance and Inductance?

- 0.011 μF
- 2 nH
- 15 mohms
Impedance of a 0.01-μF Capacitor

0.011 μF  2 nH  15 mohms

Impedance in Ohm

Frequency in MHz
What are ESR and ESL?

**Low Inductance LGA Capacitors**

**HOW TO ORDER**

- LG 1 2 3 4 5 6 Z 104 M A T 2 S 1
- High voltage: ±50V, ±200V, ±500V
- Temperature range: -55°C to 85°C
- Capacitance range: 100 nF to 1000 µF
- Lead count: 2, 3, 4, 5, 6

**CAPACITANCE RANGE**

<table>
<thead>
<tr>
<th>Device</th>
<th>Nominal 1kHz Capacitance (µF)</th>
<th>ESR (miliohms)</th>
<th>ESL (pH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LG12 0204 2T LGA</td>
<td>0.1</td>
<td>9.0</td>
<td>57</td>
</tr>
<tr>
<td>LG22 0306 2T LGA</td>
<td>1.0</td>
<td>5.0</td>
<td>35</td>
</tr>
<tr>
<td>LG32 0508 2T LGA</td>
<td>3.3</td>
<td>1.8</td>
<td>27</td>
</tr>
<tr>
<td>LGC2 0805 2T LGA</td>
<td>2.2</td>
<td>3.1</td>
<td>46</td>
</tr>
</tbody>
</table>

NOTICE: Specifications are subject to change without notice. Consult AVX's technical data sheet for the latest and most accurate information.

Actual device performance can vary based on board layout, component parasitics, and other design and device criteria. For additional information, please contact AVX at info@avx.com.
SMT Capacitor Connection Inductance

\[ C_B = 3.4 \text{nF} \]
\[ L_{\text{BULK}} = 5 \text{nH} \]
\[ C_{\text{BULK}} = 1 \mu\text{F} \]
\[ L_D = 2 \text{nH} \]
\[ C_D = 10 \text{nF} \]

Bare Board

Board with decoupling

[Graph showing impedance magnitude vs frequency for different conditions]
Types of Capacitors

- Ceramic
  - Low cost, stable, good precision

- Tantalum
  - Polarized, good energy density

- Other Electrolytic
  - Polarized, good energy density

- Mica
  - High-voltage applications
Inductors
Do Inductors Have Resistance and Capacitance?

15 mohms

160 pF

5 μH
Impedance of a 5-μH Inductor

160 pF

5 μH

15 mohms

Frequency in MHz

Impedance in Ohms
Types of Inductors

- Ferrite Core
  - High inductance in small package

- Air Core
  - Linear behavior under high-current conditions

- Common-mode
  - Impedes common-mode currents while passing differential-mode currents.
Ferrites

Fair-Rite 2643000701

![Graph showing Z, Rs, and XL vs Frequency (Hz)]
Non-Ideal Behavior of Active Devices

Currents on the lead frame of an RDR memory module at the third harmonic of the clock frequency.
Summary

- All components (when connected to a circuit) have resistance, capacitance and inductance.

- The behavior of a component at high frequencies is usually much different than the nominal (low-frequency) behavior.

- The inductance of a low-inductance device is generally determined by the connection and is not a property of the device itself.