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Survey of Current Computational Electromagnetics Techniques and Software

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I. INTRODUCTION

A. Purpose of This Report

James Clerk Maxwell's presentation titled, "A Dynamical Theory of the Electromagnetic Field" in 1864 established the foundation for electromagnetic field theory as we know it today [1]. His work, which drew upon the observations of Faraday, Ampere, Oersted, Gauss and others, introduced equations describing the relationship between electric and magnetic fields, electric charges and electric currents. From these equations, Maxwell was able to postulate the existence of electromagnetic waves, calculate the velocity of these waves and show that electromagnetic waves traveled at a velocity consistent with the velocity of light; suggesting that light could be a form of electromagnetic radiation.

The next 100 years witnessed the development of the first antennas, the wireless telegraph, radio, radar, television and digital telemetry. Yet one of the most significant breakthroughs in electromagnetic analysis, the ability to solve Maxwell's equations on a computer, did not occur until late in the 20th century.

Maxwell's equations[1] (Table 1) govern all electric, magnetic and electromagnetic behavior. They accurately describe electromagnetic behavior in any situation without making any assumptions about materials, linearity or relativity. They are the basis for all electrical interactions and they play a particularly important role in the design of antennas, EM wave propagation analysis, microwave circuit analysis, signal integrity and electromagnetic compatibility.

Table 1: Maxwell's Equations

	Integral Form	*Differential Form*
Faraday's Law:	$\oint \mathbf{E} \bullet \mathbf{dl} = -\frac{\delta}{\delta t}\left(\int_S \mathbf{B} \bullet \mathbf{ds}\right)$	$\nabla \times \mathbf{E} = -\frac{\delta \mathbf{B}}{\delta t}$
Ampere's Law:	$\oint \mathbf{H} \bullet \mathbf{dl} = \frac{\delta}{\delta t}\left(\int_S \mathbf{D} \bullet \mathbf{ds}\right) + \int_S \mathbf{J} \bullet \mathbf{ds}$	$\nabla \times \mathbf{H} = \frac{\delta \mathbf{D}}{\delta t} + \mathbf{J}$
Gauss' Law:	$\int_S \mathbf{D} \bullet \mathbf{ds} = \int_V \rho \, dV$	$\nabla \bullet \mathbf{D} = \rho$
Gauss' Magnetic Law:	$\int_S B \bullet \mathbf{ds} = 0$	$\nabla \bullet \mathbf{B} = 0$

As robust and powerful as these 4 equations are, they are virtually impossible to solve analytically for all but a small set of canonical configurations. Virtually all of the progress in this field in the 100 years following Maxwell's 1864 publication was accomplished without the

[1] The set of 4 equations commonly referred to as *Maxwell's equations* were actually derived from Maxwell's theory by Oliver Heaviside and Heinrich Hertz (independently) more than 20 years later.

ability to find exact solutions to the problems of greatest interest. As a result most antennas tended to resemble cones, rods, spheres and other canonical structures; and antenna design was a mixture of science, art and trial-and-error.

With the advent of reasonably powerful computers in the mid-1960s, a new form of electromagnetic analysis emerged; Computational ElectroMagnetic (CEM) modeling. Pioneers in this field such as Yee [2], Harrington [3], and others demonstrated that numerical solutions of Maxwell's equations could be used to accurately describe the electromagnetic behavior of real antenna configurations. For the first time, it became possible to analyze a wide range of structures and accurately determine current and field distributions without building and measuring these structures in a lab.

Of course, computers have come a long way since the 1960s, roughly doubling in speed and memory capacity every couple of years. Advances in CEM modeling techniques and software have also experienced exponential growth. A 1991 publication [4] briefly surveyed the progress in electromagnetic modeling up to that time. This report provides an expanded summary of the various EM modeling techniques and software that are currently available.

B. Categorizing CEM Modeling Tools

1. *Time vs. Frequency Domain*

All CEM modeling codes use numerical techniques to solve Maxwell's equations in some form. However, the specific technique employed, and the form of the equations solved, have a tremendous impact on the suitability of a given code to analyze a given problem. For example, some codes solve Maxwell's equations in the frequency domain (i.e. one frequency at a time), while others work in the time domain (usually calculating a system impulse response). This is not unlike circuit theory, where circuits can be analyzed in the time or frequency domains.

Frequency domain techniques tend to be more efficient when modeling problems with narrow bandwidths or high Q-factors. Time domain techniques are often more appropriate for modeling broadband structures and sometimes larger structures with complex geometries. It is interesting to note that the nature of the excitation (transient or swept-frequency) and the desired output format (function of time or frequency) are relatively minor considerations when choosing the optimum technique for a specific problem. This will become apparent as the techniques are described in more detail later in this report.

2. *IE vs. PDE*

Another important way that CEM modeling techniques are categorized is based on the form of Maxwell's equations that they solve. As indicated in Figure 1, Maxwell's equations can be expressed in two fundamental forms. The integral form of Maxwell's equations describes the behavior of fields and currents over closed loops or closed surfaces of arbitrary size and shape. For example, Gauss' Law in integral form effectively states that the net electric flux passing through any closed surface is equal to the total electric charge enclosed.

The differential form of Maxwell's equations describes the behavior of fields and currents at points in space. For example, Gauss' Law in differential form simply states that the net change in the electric flux at any point is equal to the electric charge density at that point.

The differential forms of each equation can be derived from the integral form by taking the limit as the radius of a loop or surface approaches zero. Similarly, the integral forms can be derived from the differential forms. Nevertheless, CEM modeling techniques based on solutions of the integral form of Maxwell's equations are fundamentally different from techniques based on the solution of the differential form.

Techniques based on the solution of the differential form of Maxwell's equations (Partial Differential Equation or PDE techniques) generally analyze a problem's geometry by dividing the whole structure and the space around it into small pieces. The EM interactions in each region are modeled independently. Each piece interacts only with the other pieces that are nearby. Problems that involve radiation that propagates to infinity (open geometries) must utilize an "absorbing boundary" that defines the edge of the problem space. The effectiveness of PDE techniques applied to open geometries is highly dependent on the effectiveness of the absorbing boundaries.

Integral Equation (IE) techniques generally segment only the boundaries between regions of the problem with different electromagnetic properties. For example, in a 3D structure, the "pieces" would be 2D surface patches or 1D wire segments.

IE techniques generally require fewer pieces than PDE techniques; however every piece in an IE technique usually interacts with every other piece no matter where it is located, so the amount of calculation involved to solve the problem is not necessarily lower.

3. 2D vs. 3D

CEM modeling techniques can be implemented in 1, 2, or 3 dimensions. 1-dimensional codes are generally only of interest to students and academicians, though they can be useful in specialized applications. 2D codes are commonly used to obtain the per-unit-length transmission line parameters corresponding to circuit board trace geometries or cable cross-sections. They can also be used to model certain structures with symmetry in one dimension (e.g. a biconical antenna). Although most 2D problems could theoretically be analyzed using a 3D code (for example by modeling a length of cable and determining the fields in a single cross-section), this is rarely done. 2D modeling is much more efficient than 3D modeling (e.g. N^2 vs. N^3 unknowns corresponding to N^6 vs. N^9 calculations) and 2D codes optimized for a particular application will generally be much easier to use and more accurate than similar 3D codes.

Most problems of interest however, are 3-dimensional and require a 3-dimensional modeling code. "Full wave" modeling codes calculate time-varying fields in 3 independent dimensions without placing any special restrictions on the behavior of the fields in any given dimension. All of the CEM modeling techniques described in this report can be implemented in 2 or 3 dimensions. However, 2D and 3D implementations of a given technique can be very different. It is not necessarily true that the best technique for analyzing a particular 2D problem will be the best technique for analyzing a "similar" 3D problem.

4. Static, Quasi-static, Full-wave or Asymptotic

Claims that a particular technique works "from DC to daylight" are not uncommon, but CEM modeling codes are only accurate and efficient over a limited band of frequencies. Actually, it is not the frequency itself that matters as much as the size of the structure to be modeled relative to the wavelengths of the analysis.

Full wave modeling techniques, which are the main focus of this report, are most effective when the size of the structure being analyzed is within an order of magnitude of one wavelength (e.g. 0.1λ to 10λ). Smaller objects are often more readily modeled with static or quasi-static codes. Larger objects are often better suited for analysis using asymptotic techniques.

There are two categories of static modeling codes: *electrostatic* and *magnetostatic*. Since the electric and magnetic fields are uncoupled at DC, an electrostatic solver yields no information about the magnetic fields. Similarly, a magnetostatic solver does not calculate electric field distributions. The sources in an electrostatic analysis will be DC voltages or static charge distributions. All materials behave like perfect conductors or perfect dielectrics. Electrostatic solvers are used to determine induced charges or voltages, electric field distributions, or capacitances. The sources in a magnetostatic analysis are DC current distributions. Magnetostatic solvers are used to determine induced currents, magnetic field patterns or inductances.

Static field solvers do not employ time or frequency domain techniques, because neither time nor frequency is variable. Static solvers can be either IE or PDE, however, the primary difference being whether the entire volume or just the material interfaces are sectioned.

Static field analysis requires much less computation and tends to be more stable than full-wave analysis. For these reasons, static field solvers are capable of analyzing configurations with significantly greater complexity than full-wave solvers using similar computer resources.

Quasi-static modeling codes generally employ static field solvers with some allowance made for the solution to vary (slowly) with time. For example, an analysis of an electric motor may employ a magnetostatic solver to recalculate the magnetic field over and over as the rotor changes position. Similarly, an analysis of dielectric breakdown in a capacitor may rely on an electrostatic field solver while adjusting static charge distributions in response to the calculated field strengths and non-linear properties of the dielectric.

At high frequencies, where the structure being analyzed is many wavelengths, full-wave solutions are less attractive because the number of electrically small "pieces" required to represent the entire problem space can quickly grow to a number that exceeds the computer resources available. At these frequencies, codes employing asymptotic techniques are usually more efficient and accurate. Asymptotic techniques model radiated fields as if they were rays of light; emanating from a source and traveling in a straight line to the point where the field is calculated. These rays may be absorbed, reflected or diffracted by objects in the problem space. The calculated field at a given point is the vector sum of all rays reaching that point from all sources. There is no need to grid the entire volume as in full-wave PDE techniques; and there is not an interaction between every pair of elements as in IE-based full-wave methods.

5. *Circuit-based Field Solvers*

There are at least two CEM modeling techniques that convert a field problem to an RLC circuit analysis problem that can then be solved using SPICE-like circuit modeling tools. An advantage of these techniques is that the resulting circuit can be analyzed in either the time or frequency domain. Another advantage is that these techniques can make it easier to incorporate linear or non-linear circuit elements into the field analysis.

6. *Linear vs. Higher-order Methods*

Since virtually all general purpose CEM modeling codes break the problem space into a set of pieces (or elements), it is useful to categorize techniques depending on how the charge, current or field quantities within these elements are defined. Most CEM modeling tools employ linear elements, meaning that the quantities within a single element are described by a linear function whose entire domain is confined within the boundaries of that element. Specifying the amplitude of that function completely determines the quantity within the element independent of the amplitudes specified outside the element.

Higher-order techniques employ elemental functions that are not confined to the boundaries of a single element. These functions are normally nth-order polynomials where n>1. This formulation allows complex charge, current or field distributions to be represented accurately with fewer (i.e. larger) elements. Fewer elements generally translates to less computation time and memory, so the primary advantage of higher-order techniques is that they solve problems more quickly and accurately than their linear counterparts.

The primary disadvantage of higher-order techniques is that they can require significantly more effort to segment properly and the additional "bookkeeping" required to set up a problem for analysis can sometimes consume the resources that were saved by using the higher-order formulation.

Virtually all of the full-wave modeling techniques discussed in this report can be formulated with higher-order elements, but moment method and finite element codes are perhaps better suited for this than codes based on other techniques.

C. Outline of This Report

The next five sections of this report describe various CEM modeling techniques commonly employed by commercial software or used to model specific problems of interest. The focus of these sections is on full-wave EM modeling, though several of these techniques can also be formulated to solve quasi-static problems. Section VII reviews high-frequency (asymptotic) techniques. Section VIII discusses hybrid methods, which combine two or more techniques to create codes that can analyze a wider variety of problem geometries. Finally, Section IX provides a brief summary of commercially available CEM modeling software available as of the date of this report.

II. The Method of Moments

In the 1960s, R.F. Harrington [5] and others applied a technique called the *Method of Moments* to the solution of electromagnetic field problems. The Method of Moments (also called the Method of Weighted Residuals) is a technique for solving linear equations of the form,

$$\mathcal{L}(\phi) = f \tag{II-1}$$

where $\mathcal{L}(\bullet)$ is a linear operator, f is a known excitation or forcing function, and ϕ is an unknown quantity. To solve this problem on a digital computer, we start by expressing the unknown solution as a series of *basis* or *expansion* functions, υ_n,

$$\phi = \sum_{n=1}^{N} a_n v_n \qquad \text{(II-2)}$$

where a_n are unknown coefficients describing the amplitude of each term in the series.

Now instead of one equation with a continuous unknown quantity, f, we have an equation with N scalar unknowns,

$$\mathcal{L}(a_1 v_1 + a_2 v_2 + \ldots + a_N v_N) = f \qquad \text{(II-3)}$$

To solve for the values of a_n, we need N linearly independent equations; so N different *weighting* or *testing* functions, w_n, are applied. This yields the following system of N equations in N unknowns:

$$\begin{aligned} &< w_1, \mathcal{L}(\phi) > = < w_1, f > \\ &< w_2, \mathcal{L}(\phi) > = < w_2, f > \\ &\qquad\qquad \vdots \\ &< w_N, \mathcal{L}(\phi) > = < w_N, f > \end{aligned} \qquad \text{(II-4)}$$

Or, expressed in matrix form,

$$\begin{bmatrix} < w_1, \mathcal{L}(v_1)) > & < w_1, \mathcal{L}(v_2)) > & \cdots & < w_1, \mathcal{L}(v_N)) > \\ < w_2, \mathcal{L}(v_1)) > & < w_2, \mathcal{L}(v_2)) > & \cdots & < w_2, \mathcal{L}(v_N)) > \\ \vdots & \vdots & & \vdots \\ < w_N, \mathcal{L}(v_1)) > & < w_N, \mathcal{L}(v_2)) > & \cdots & < w_N, \mathcal{L}(v_N)) > \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} < w_1, f_1 > \\ < w_2, f_2 > \\ \vdots \\ < w_N, f_N > \end{bmatrix} \qquad \text{(II-5)}$$

This linear system of equations has the form,

$$[Z][A] = [B] \qquad \text{(II-6)}$$

where the elements of $[Z]$ are known quantities that can be calculated from the linear operator, $\mathcal{L}(\bullet)$, and the chosen basis and weighting functions. The elements of $[B]$ are determined by applying the weighting functions to the known forcing function. The unknown elements of $[A]$ can be found by solving the matrix equation. After solving for $[A]$ (i.e. the unknown coefficients, a_n), the value of ϕ is determined using Equation (II-2).

The Method of Moments (MoM) can be used to solve a wide range of equations involving linear operations including integral and differential equations. This numerical technique has many applications other than electromagnetic modeling; however the MoM is widely used to solve equations derived from Maxwell's equations. In general, moment method codes generate and solve large, dense matrix equations and most of the computational resources required are devoted to filling and solving this matrix equation. The particular form of the equations that is solved and the choice of basis and weighting functions have a great impact on the size of this matrix and ultimately the suitability of a given moment method code to model a given geometry.

A. Equation Options

The most common equation form solved by CEM modeling codes based on the Method of Moments is the *Electric Field Integral Equation* (EFIE). This is an equation of the form,

$$\mathbf{E} = f_e(\mathbf{J}, \mathbf{M}) \tag{II-7}$$

where $\mathbf{E}$ is the impressed (i.e. source) electric field and $\mathbf{J}$ and $\mathbf{M}$ are the induced electric and magnetic current densities, respectively. The EFIE will be discussed further in the section describing the Boundary Element Method. Generally, codes that solve a form of the EFIE excel at modeling open (unbounded) geometries in which the electric field dominates in the near-field region of the source.

Another equation solved by Moment Method codes is the *Magnetic Field Integral Equation* (MFIE), which has the general form,

$$\mathbf{H} = f_m(\mathbf{J}, \mathbf{M}) \tag{II-8}$$

where $\mathbf{H}$ is the impressed (i.e. source) magnetic field intensity. Codes that solve a form of the MFIE are best suited for modeling geometries with circulating currents, where the magnetic near field is dominant.

Moment Method codes based on the EFIE or MFIE alone, may exhibit unstable behavior when the modeling surfaces form a resonant cavity at a particular frequency. To avoid this, many moment method codes solve a linear combination of the EFIE and MFIE known as a *Combined Field Integral Equation* (CFIE). This requires more calculations to fill the matrix, but results in a more stable solution when the modeling surface is large enough to support an interior resonance.

Some CEM modeling codes employ the Method of Moments to solve other equations. For example, static modeling codes often solve a form of Laplace's equation relating electric field strengths to charge densities or magnetic field strengths to current densities. The Generalized Multiple Technique (GMT), which is described in another section of this report, employs a moment method to solve equations for the electric field generated by multipole sources.

B. Basis and Weighting Functions

An appropriate choice of basis and weighting functions can make a tremendous difference in the number of elements, N, required to obtain an accurate solution. Since the solution is represented as a summation of basis functions [see Eq. (II-2)], it is important to choose basis functions that accurately represent the solution with a small number of terms. For example, when solving for the current distribution on a surface, the basis functions should be current distribution elements that can be summed together in a way that is able to efficiently approximate any overall current distribution that might result from the analysis.

Weighting functions should be chosen that maximize the linear independence of the various weighted forms of the equation. Often, the best choices of weighting functions are functions that are identical to the basis functions. Moment method techniques that employ identical basis and weighting functions are called *Galerkin* techniques.

III. THE BOUNDARY ELEMENT METHOD

Boundary Element Method (BEM) codes use the method of moments to solve an EFIE, MFIE or CFIE for electric and/or magnetic currents on the surfaces forming the interfaces between any two dissimilar materials. Most CEM modeling codes that bill themselves as simply "moment method" codes employ a boundary element method.

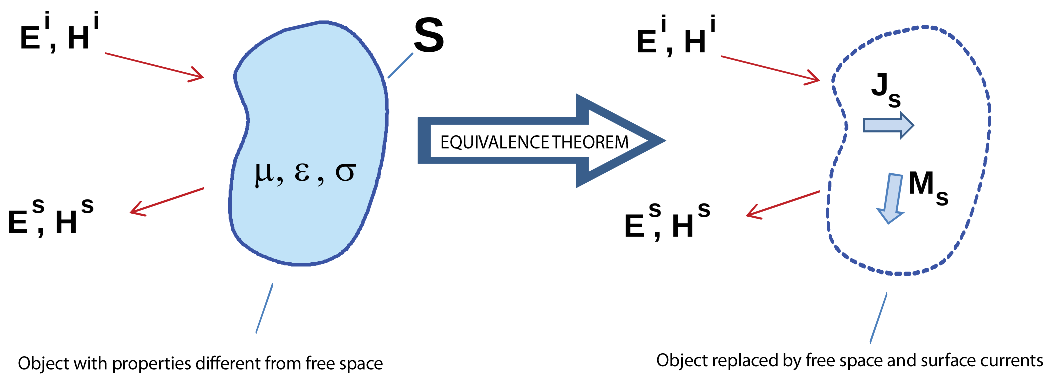

Figure 1: Principle of Equivalence.

The first step in a boundary element analysis is to represent the problem geometry as a distribution of equivalent surface currents in a homogeneous medium (usually free space). As illustrated in Figure 1, the fields exterior to an object consist of fields incident on the object, fields reflected from the object and fields emanating from the object. The Equivalence Theorem [6] states that any field distribution exterior to an object can be exactly duplicated by removing the object and replacing it with a set of equivalent electric and magnetic currents on the boundary surface.

Since the forms of EFIE and MFIE used by boundary element methods are only valid for current distributions in a uniform homogeneous medium, all objects in the problem space must be removed and replaced with (initially unknown) surface currents conforming to their boundaries.

Equations (III-1) and (III-2) below show a common form of the EFIE and MFIE employed by boundary element method codes that model metallic objects only (i.e. there are no equivalent magnetic surface currents);

$$\mathbf{E}(r) = \frac{-j\eta}{4\pi k}\int_S \mathbf{J}_S(r')\bullet\mathbf{G}_e(r,r')dS' \qquad \text{(III-1)}$$

$$\mathbf{H}(r) = \frac{1}{4\pi}\int_S \mathbf{J}_S(r')\times\nabla'\mathbf{G}_m(r,r')dS' . \qquad \text{(III-2)}$$

In these equations, $\int_S dS'$ represents integration over all boundary surfaces. Note that since the boundary only exists at places where an interface between two different materials occurred, the size of the boundary is limited (i.e. there is no integration to infinity).

The functions $\mathbf{G_e}$ and $\mathbf{G_m}$ in these equations relate source currents to the electric and magnetic field generated by those currents, respectively. $\mathbf{G_e}$ and $\mathbf{G_m}$ are called *Green's functions* [7] and they play a central role in boundary element analysis. Free-space Green's functions express the field emanating from the surface current represented by an individual basis function (e.g. the current on an individual surface patch). However, other Green's functions can be employed to express the field emanating from more complex structures that are common to a particular problem geometry. For example, a geometry consisting of metal surfaces coated with a thin dielectric may employ a special Green's function that expresses the fields emanating from the combined surfaces of the metal-dielectric and dielectric-air interfaces. This can substantially reduce the number of surface elements required to model the problem.

All Green's functions are approximate expressions that are accurate for a limited range of frequencies, distances and source geometries. Many boundary element methods employ several Green's functions to model different regions of a problem or different problem environments.

General purpose 3D BEM codes usually employ basis and weighting functions that are linear current distributions on a rectangular or triangular surface patch. There are generally two unknowns per patch corresponding to two orthogonal current vectors. Thin wires can be represented very efficiently with a single unknown representing the amplitude of the current distribution on each wire segment.

Point matching techniques employ basis and weighting functions that are simple impulse functions often at the center of each patch or segment. Pulse matching techniques employ basis and weighting functions that have a constant value everywhere on the patch or segment. More accurate implementations employ basis and weighting functions that transition smoothly from one patch to the next. Rao-Wilton-Glisson (RWG) [8] basis functions are a popular choice for codes that employ triangular surface patch elements. Roof-top basis functions [9] are often employed by codes that use rectangular elements.

Generally, CEM software employing a boundary element method excels at modeling unbounded problems, particularly when it is not necessary to model regions of great complexity in detail. Structures that can be adequately represented with a wire grid can be analyzed very effectively using boundary element methods, because these methods model wires very efficiently.

Table 2 lists various strengths and weakness of BEM modeling techniques. Note that the capabilities of any particular modeling software depend strongly on the form of the integral equation solved, the choice of basis and weighting functions, the Green's function(s) employed, and the matrix solver and any optimization techniques employed.

Table 2: Strengths and Weaknesses of the Boundary Element Method

BEM Modeling Strengths	***BEM Modeling Weaknesses***
– **Excellent for modeling unbounded (radiation) problems.** – **Excellent for modeling metal plates and thin wires.** – **Good for modeling structures with lumped circuit elements included.**	– Does not model inhomogeneous or complex materials well. – Not good for modeling problems that combine small detailed geometries with larger objects. – CFIE formulation required to model enclosed structures of resonant size.

Table 3 lists various CEM modeling codes that are based on a boundary element method. The codes listed and the comments in Table 3 are based on the information available to the authors as of the publication date of this report.

Table 3: CEM Modeling Codes that use the Boundary Element Method

Software Title	***Description***	***Source***
Amperes/Coulomb/Faraday	3D quasi-static	Integrated Eng. Software
Antenna Model	wire structures	Teri Software
CableMod	2D cable modeling	CST/Simlab
CONCEPT-II	3D full-wave	TU Hamburg-Harburg
Efield FD start-pack	3D full-wave	efield
Electro/Oersted	2D quasi-static	Integrated Eng. Software
EM3DS	MMIC Modeling	MEM Research
EZNEC Pro	Wire structures	EZNEC
FEKO	3D full-wave	EM Software and Systems
GEMACS	3D full-wave (wires and plates)	Advanced Electromagnetics
NEC2	3D full-wave (wires and plates)	Open Source
OPERA 2D	2D quasi-static	Vector Fields
OPERA 3D	3D quasi-static	Vector Fields
PCBMod	2D circuit board structures	CST/Simlab
SuperNEC	Wire structures	Poynting Antennas
WIPL-D Pro	3D full-wave	WIPL-D

IV. THE FINITE ELEMENT METHOD

Scalar finite element methods have been used by civil and mechanical engineers to analyze material and structural problems since the 1940s. However it wasn't until the 1960s that FEM codes were developed to solve problems in electromagnetics. Some of the pioneers in this field were Silvester [10], Zienkiewicz [11], and Wexler [12]. Initial FEM-based CEM modeling codes were applied to problems in electrostatics and magnetostatics. Later they were used to solve high-frequency problems in 2 dimensions. Practical 3-dimensional codes did not appear until the 1980s due largely to problems with vector parasites [13, 14] and unreliable absorbing boundary conditions [15]. Unwanted reflections from absorbing boundaries continue to be a problem with full-wave 3D FEM codes even today.

Like BEM techniques, finite element methods can be based on different formulations (even the method of moments). However BEM techniques always solve an integral equation and FEM techniques always solve a differential equation. Every FEM code divides the entire problem domain into small elements. For 2D problems the elements are usually triangles or rectangles. For 3D problems, the elements are usually tetrahedra (4 faces) or bricks (6 faces). The domain must be finite and bounded. Modeling an unbounded (e.g. radiation) problem requires that the problem domain be bounded with special elements that absorb all incident energy. These elements are called ABC (Absorbing Boundary Condition) elements.

The unknowns in *scalar* FEM codes are the three orthogonal components of the field at the "nodes" (vertices) of each element. The unknowns in *vector* FEM codes are the field components along the edges of each element. Scalar codes are conceptually simpler, but they are unsuitable for full-wave modeling, because they are susceptible to spurious solutions that can cause significant and unpredictable errors in the solution. Vector FEM codes are much less likely to exhibit these parasitics.

To form a linear system of equations, the governing differential equation and associated boundary conditions are converted to an integro-differential form using either a variational method or a weighted-residual (moment) method. Variational methods solve for the unknown quantity by minimizing an energy functional. Weighted-residual methods multiply a weak form of Maxwell's equations by a weighting function and integrate over each element. Ultimately, a matrix equation is generated in the form,

$$[A][X]=[B] \tag{IV-1}$$

where $[X]$ is a vector of the unknown field quantities, $[B]$ is a vector of source terms, and $[A]$ is a sparse matrix whose only non-zero values correspond to positions in the matrix corresponding to edges that share an element.

Generally, the matrices generated by FEM codes are must larger than the matrices generated by BEM codes applied to similar geometries. This is because gridding an entire problem volume requires many more elements than gridding just the material interfaces. However, because FEM matrices are very sparse, they do not necessarily require more storage or computing resources to solve than the small, but dense, matrices generated by BEM codes.

As indicated previously, modeling unbounded problems requires special absorbing elements (ABCs). Many formulations of these elements have been proposed [15-22]. The ABCs that have been developed for 2D FEM codes work very well; however 3D FEM ABCs work well only at prescribed angles of incidence resulting in the need to locate the boundaries sufficiently far from other structures. Hybrid FEM/BEM codes terminate open surfaces of the FEM volume with a BEM surface negating the need for ABCs [23]. Unfortunately, the BEM portion of the resulting matrix is dense, which can significantly increase the amount of computational resources required.

Perhaps the most attractive feature of the finite element method is its ability to model configurations that have complicated geometries and incorporate various materials. The electrical properties of each element are defined independently and elements can be as small or as large as needed to facilitate the analysis.

Table 4 lists various strengths and weakness of FEM modeling techniques. Note that the capabilities of any particular modeling software depend on the specific formulation, the matrix solver and any optimization techniques employed.

Table 4: Strengths and Weaknesses of the Finite Element Method

FEM Modeling Strengths	***FEM Modeling Weaknesses***
– **Excels at modeling inhomogeneous or complex materials** – **Excels at modeling problems that combine small detailed geometries with larger objects** – **Excels at modeling structures in resonant cavities or waveguides**	– Absorbing boundary required for modeling unbounded (radiation) problems – Difficult to model thin wires accurately

Table 5 lists various CEM modeling codes that are based on the finite element method. The codes listed and the comments in Table 5 are based on the information available to the authors as of the publication date of this report.

Table 5: CEM Modeling Codes that use the Finite Element Method

Software Title	***Description***	***Source***
Amaze (HiPhi/Magnum)	3D quasi-static	Field Precision
Analyst	2D and 3D full-wave	STAAR
Microwave Studio –FDS	3D full-wave	CST
EMDS	3D full-wave	Agilent Technologies
EMS	3D quasi-static	ElectromagneticWorks, Inc.
FEMM	2D quasi-static	Dr. David Meeker
Flux2D/3D	2D and 3D quasi-static	Magsoft
HFSS	3D full-wave	Ansys / Ansoft
JCMsuite	optics	JCMwave
MagNet / ElecNet	3D quasi-static	Infolytica
Magneto / Oersted	2D quasi-static	Integrated Eng. Software
Maxwell	2D/3D quasi-static	Ansys / Ansoft
Opera 2D/3D	2D/3D quasi-static	Vector Fields
Q3D Extractor	2D/3D quasi-static	Ansys / Ansoft
QuickField	2D quasi-static	Tera Analysis
Tricomp EStat/PerMag	2D quasi-static	Field Precision
Tricomp WaveSim	2D high-frequency	Field Precision

V. THE FINITE DIFFERENCE TIME DOMAIN METHOD

The Finite Difference Time Domain (FDTD) method, as first proposed by Yee [2], is a direct solution of Maxwell's time dependent curl equations. It uses simple central-difference approximations to evaluate the space and time derivatives. A basic element of the FDTD space lattice is illustrated in Figure 2. An electric-field grid is offset from a magnetic-field grid in both space and time. A first-order central-difference approximation can be expressed as,

$$\frac{1}{\Delta l}\left[E_{z1}(t)+E_{y2}(t)-E_{z3}(t)-E_{y4}(t)\right]=-\frac{\mu_0}{2\Delta t}\left[H_{x0}(t+\Delta t)-H_{x0}(t-\Delta t)\right] \qquad \text{(V-1)}$$

where Δl is the length of one side of the cubical cell in Figure 2. $H_{xo}(t+\Delta t)$ is the only unknown in this equation, since all other quantities were found in a previous time step. In this way, the electric field values at time t are used to find the magnetic field values at time $t+\Delta t$. A similar central-difference approximation of Equation (V-1) can then be applied to find the electric field values at time $t+2\Delta t$ from the magnetic field values at time $t+\Delta t$. By alternately calculating the electric and magnetic fields at each time step, fields are propagated throughout the grid.

Time stepping is continued until a steady state solution or the desired response is obtained. The required computer storage and running time is proportional to the electrical size of the volume being modeled and the grid resolution.

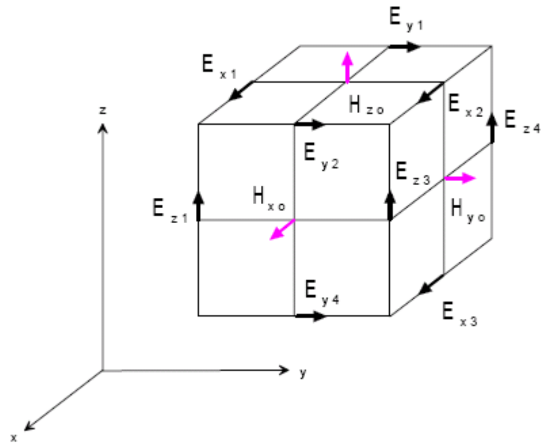

Figure 2: Basic Element of the FDTD Space Lattice.

For an open region problem, an absorbing boundary condition (ABC) is used to truncate the computational domain. One technique, which is obtained by factoring the wave equation to permit only outgoing waves, is differential based ABCs, such as those proposed by Engquist [24], Lindman [25], Mur [26], Liao [27], Keys [28], and Higdon [29]. Another is material based ABCs that are constructed so that the fields are dampened as they propagate into an absorbing medium. Rappaport [30] proposed an ABC employing pyramid-shaped absorber material. In 1994, Berenger [31] introduced the perfectly matched layer (PML) absorbing boundary condition. This ABC outperforms any that had been proposed previously and is widely used today. Andrew [32] compared the accuracy of the Berenger perfectly matched layer and the Lindman higher-order ABCs for the FDTD method. Accuracy studies of ABCs have also been conducted for dispersive media [33, 34]. In 2003, Diaz [35] introduced a new radiation boundary condition for FDTD based on self-teleportation of fields.

Because the basic elements are cubes, curved surfaces on a scatterer must be staircased. For many configurations this does not present a problem. However for configurations with sharp, acute edges, this approximation may lead to significant errors [36-37], and an adequately staircased approximation may require a very small grid size. Surface-conforming FDTD techniques with non-rectangular elements have been introduced to combat this problem [38-45].

Since all of the elements in an FDTD analysis must generally be the same size, the size of the elements is determined by the smallest structural details that need to be modeled. If an object under consideration contains small-scale geometries, such as a narrow slot or a very thin wire, an excessively fine grid would have to be used to accurately model the associated fields. To overcome these shortcomings, sub-cellular structures [46-55] have been introduced. Sub-cellular structures are essentially special FDTD cells whose boundary conditions have been altered to model small structures contained within the cells.

One major advantage of the FDTD method is the ability to obtain wideband results using a transient excitation in one simulation. Frequency domain results can be obtained by applying a discrete Fourier transform to the time domain results. Since many materials have frequency dependent properties, it is necessary to take special precautions to model these materials correctly with a time-domain technique. In 1990, Luebbers [56] used a recursive convolution scheme to model a Debye media; this was the first frequency dependent FDTD formation. Kashiwa and co-workers [57-59] published the first papers utilizing the auxiliary differential equation (ADE) method to model Debye media, Lorentz media, and media obeying the Cole-Cole Circular Arc law. In 1992, Sullivan [60] proposed a dispersive formulation based on Z transforms. Petropoulos [61] provided a comparison of the stability and phase error among frequency dispersive FDTD methods.

Lossy surfaces can be modeled in FDTD codes by utilizing a surface impedance boundary condition (SIBC) [62-64]. A thin material sheet model has also been developed for the FDTD method [65-67].

A primary advantage of FDTD methods is their great flexibility. Arbitrary signal waveforms can be modeled as they propagate through complex configurations of conductors, dielectrics, and lossy non-linear non-isotropic materials. Another advantage of FDTD techniques is that they are readily implemented on massively parallel computers, particularly vector processors and SIMD (single-instruction-multiple-data) machines.

Time stepping techniques like FDTD are subject to dispersion errors when the time step is too large for the given problem size. Many researchers have studied the numerical dispersion error inherent in the FDTD method [68-76], but it is easily controlled by using appropriately small time steps.

Table 6 lists various strengths and weakness of FDTD modeling techniques. Table 7 lists various CEM modeling codes that employ an FDTD solver.

Table 6: Strengths and Weaknesses of the Finite Difference Time Domain Method

FDTD Modeling Strengths	***FDTD Modeling Weaknesses***
– **Excels at modeling inhomogeneous or complex materials** – **Excels at modeling very large problems** – **Runs efficiently on highly parallel computers**	– Absorbing boundary required for modeling unbounded problems, but PML boundaries work very well. – Difficult to model thin wires – Uniform cells must be small enough to model necessary detail, but still fill the entire volume. – High Q structures are not modeled efficiently

Table 7: CEM Modeling Codes that use the Finite Difference Time Domain Method

Software Title	***Description***	***Source***
ApsimFDTD	3D full-wave	Applied Simulation Tech.
CST M. Studio - TS	3D full-wave	CST
EMA3D	3D full-wave	Electromagnetic Appl.
EMPIRE XCcel	3D full-wave	Empire
EZ-FDTD	3D full-wave	EMS-Plus
Fidelity	3D full-wave	Zeland Software
GEMS	3D full-wave	2COMU
LC	3D full-wave	Cray Research
PAM-CEM	3D full-wave	ESI Group
SEMCAD X	3D full-wave	Schmid & Partner Eng.
Toy	3D full-wave	The CEMTACH Group
XFDTD	3D full-wave	Remcom

VI. OTHER POPULAR CEM MODELING TECHNIQUES

A. The Finite Integration Technique

The Finite Integration Technique (FIT) is a consistent formulation for the discrete representation of Maxwell's equations on spatial grids. First proposed by Weiland [77] in 1977, the finite integration technique can be viewed as a generalization of the FDTD method. It is also similar to the finite element method.

Weiland [77, 78] proposed exact algebraic analogues to Maxwell’s equations that guarantee physical properties of computed fields and lead to a unique solution. By discretizing the integral form of Maxwell’s equations on a pair of dual interlaced discretization grids, the finite integration technique generates so-called Maxwell’s Grid Equations (MGEs) that guarantee the physical properties of computed fields and lead to a unique solution.

$$Ce = -\frac{d}{dt}b \qquad \text{(VI-1)}$$

$$\tilde{C}h = \frac{d}{dt}d + j \qquad \text{(VI-2)}$$

$$Sb = 0 \qquad \text{(VI-3)}$$

$$\tilde{S}d = q \qquad \text{(VI-4)}$$

where e is the electric voltage between the grid points and h is the magnetic voltage between dual grid points. d , b and j are fluxes over grid or dual grid faces. The allocation of the voltage and flux components on the dual grids is shown in Figure 3.

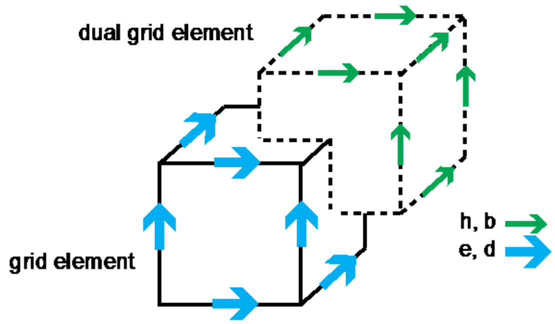

Figure 3: Allocation of the voltage and flux components in the mesh.

Due to the consistent transformation, the analytical properties of the fields are maintained resulting in corresponding discrete topological operators on the staggered grid duplet. The topology matrices C , $\tilde{C}$, S and $\tilde{S}$ correspond to the curl- and the div- operators. The tilde means that the operator is performed on the dual grid.

After discretization, the material property relations become

$$d = M_{\varepsilon} e \tag{VI-5}$$

$$b = M_{\mu} h \tag{VI-6}$$

$$j = M_{\kappa} e + j_A \tag{VI-7}$$

where M_ε, M_μ and M_k are matrices describing the material properties. The relations in (VI-1) - (VI-4) are exact on a given mesh, however, the material matrices contain the unavoidable approximations of any numerical procedure. In addition, these matrices have diagonal form [78].

Employing a so-called leap-frog scheme which samples values of e and h at times separated by a half time step, the MGEs can be rewritten as a set of two recursion formulas:

$$h^{i+1} = h^{i} - \Delta t M_{\mu}^{-1} C e^{i+1/2} \tag{VI-8}$$

$$e^{i+3/2} = e^{i+1/2} + \Delta t M_{\varepsilon}^{-1} \left(\tilde{C} h^{i+1} - j^{i+1} \right) \tag{VI-9}$$

The recursion is stable if the time step inside an equidistant grid is restricted by the Courant criterion to

$$\Delta t \le \frac{1}{c\sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}} \tag{VI-10}$$

The calculation of each further time step only requires one matrix-vector multiplication. Thus it has the advantage being an explicit algorithm.

The FIT can be applied to different mesh types [79, 80]. On Cartesian grids, the time-domain FIT is equivalent to FDTD.

B. The Partial Element Equivalent Circuit Method

The Partial Element Equivalent Circuit (PEEC) Method was developed by Ruehli [81, 82] in the 1970s and 1980s. This electromagnetic modeling technique does not solve Maxwell's equations directly. Instead, it models electric field interactions as capacitances and magnetic field interactions as inductances. Metallic geometries are segmented on their surfaces, much as they would be using other integral equation techniques. Dielectrics (other than free space) are segmented throughout their volume. Each segment is then represented as a node in a large circuit model. The currents flowing in these segments are determined by analyzing the circuit using a SPICE-like circuit solver.

This technique has the advantage that it is reasonably intuitive to many electrical engineers and it is easy to integrate the field solver with real circuit elements. Problems can be solved in the time domain or in the frequency domain,

Static modeling with PEEC codes is particularly powerful and intuitive [83-86]. Full-wave modeling requires a circuit solver that can handle time-retarded values of mutual capacitance and inductance. Since this is not a normal feature of SPICE solvers, the circuit solver for full-wave PEEC codes must generally be distributed with the PEEC software.

C. The Transmission Line Matrix Method

The Transmission Line Matrix (TLM) method, introduced by Johns [87], is similar to the FDTD method in terms of its capabilities, but its approach is unique. Like FDTD, analysis is performed in the time domain and the entire region of the analysis is gridded. Instead of interleaving E-field and H-field grids however, a single grid is established and the *nodes* of this grid are interconnected by virtual transmission lines. Excitations at the source nodes propagate to adjacent nodes through these transmission lines at each time step.

The symmetrical condensed node formulation introduced by Johns [88] has become the standard for three-dimensional TLM analysis. The basic structure of the symmetrical condensed node is illustrated in Figure 4. Each node is connected to its neighboring nodes by a pair of orthogonally polarized transmission lines.

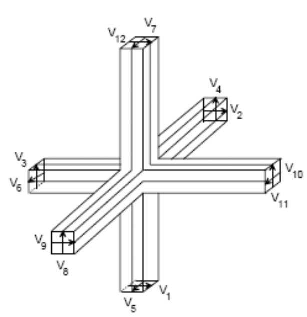

Figure 4: The Symmetrical Condensed Node.

Absorbing boundaries are easily constructed in TLM meshes by terminating each boundary node transmission line with its characteristic impedance. Simons and Bridges [89, 90] derived 2D TLM absorbing boundaries, Saguet [91] proposed matched load simulation based on a Taylor series expansion technique for 2D TLM waveguide problems. Morente et al. [92] investigated TLM absorbing boundaries based on a one-way equation technique for three-dimensional problems. Chen [93] implemented absorbing and connecting boundary conditions into a 3D TLM simulation based on Higdon's absorbing conditions, a Taylor expansion algorithm, and connecting boundary conditions. Kukutsu [94] presented a super absorption boundary condition for guided waves. Eswarappa [95] and Pena [96] describe an algorithm that interfaces the three-dimensional (3D) transmission-line matrix (TLM) with an absorbing-boundary condition (ABC) based on the perfectly matched-layer (PML) approach. Shao [97] implemented a Z-transform based absorbing boundary condition.

Generally, dielectric loading is accomplished by loading the nodes with reactive stubs. These stubs are usually half the length of the mesh spacing and have a characteristic impedance appropriate for the amount of loading desired. Lossy media can be modeled by introducing loss into the transmission line equations or by loading the nodes with lossy stubs. De Menenes [98-100] presented the modeling of a nonlinear Lorentz dielectric and a frequency independent dielectric with a Kerr nonlinearity. Hein [101, 102] developed the TLM model for propagation in both magnetized plasma and ferrite. Paul [103-105] designed TLM algorithms for one-dimensional (1D) and three-dimensional (3D) models involving linear frequency-dependent isotropic dielectric media, anisotropic materials, and frequency dependent nonlinear dielectric materials.

The strengths of the TLM method are similar to those of the FDTD method. Complex, nonlinear materials are readily modeled. Impulse responses and the time-domain behavior of systems are determined explicitly. And, like FDTD, this technique is suitable for implementation on massively parallel machines.

Both the TLM and FDTD techniques are powerful and widely used. For many types of EM problems they represent the only practical methods of analysis. Deciding whether to utilize a TLM or FDTD technique is often based on personal preference. Many engineers find the transmission line analogies of the TLM method to be more intuitive and easier to work with. On the other hand, others prefer the FDTD method due to its simple, direct approach to the solution of Maxwell's field equations. For modeling propagation in complex materials, TLM may offer a more straightforward solution than FDTD. Also, the TLM method generally does a better job of modeling complex boundary geometries, because both **E** and **H** are calculated at every boundary node. However, TLM methods require more computer memory per node than FDTD.

D. The Finite Volume Time Domain Technique

The Finite Volume Time Domain (FVTD) method was first applied to electromagnetic problems in the early 1990's [106, 107]. This technique is based on Maxwell's curl equations in their conservative form [108],

$$-\frac{\partial}{\partial t}\iiint_V \vec{B}d\upsilon = \oiint_{\partial V} \vec{n}\times\vec{E}da \qquad \text{(VI-11)}$$

$$\frac{\partial}{\partial t}\iiint_{V}\overrightarrow{D}d\upsilon=\oint\!\!\!\oint_{\partial V}\vec{n}\times\overrightarrow{H}da \qquad \text{(VI-12)}$$

where ∂V represents the boundary enclosing V. The FVTD method solves the above form of Maxwell's equations numerically by integration over small elementary volumes. Because there are no limitations for selecting the shape of the elementary volumes, the FVTD is well suited for implementation with unstructured meshes. It has become a powerful alternative to the finite difference time domain (FDTD) method for electromagnetic problems where conformal meshing is advantageous.

Like FDTD, FVTD methods can take advantage of PML absorbing boundaries. Bonnet [109] presented a vertex-centered FVTD model of the PML for scattering problems. Sankaran [110] extended the PML concept to the cell-centered FVTD approach and systematically characterized its performance using both structured and unstructured finite volume meshes. He introduced a uniaxial Maxwellian absorber using PML to solve the waveguide truncation problem [111] and extended the absorber to incorporate radial anisotropy for modeling cylindrical geometries [112]. Fumeaux [113] presented a spherical perfectly matched absorber for finite-volume 3D domain truncation. Pinto [114] incorporated the uniaxial perfectly matched layer in the analysis of light propagation in photonic bandgap devices.

The FVTD method is a promising numerical technique with good potential for the simulation of a variety of complex electromagnetic problems [115-118]. Bonnet [119] presented a method for the resolution of electromagnetic diffraction by complex structures; results obtained for an aircraft were compared with results from a classical FDTD code. Lacour [120] described a multi-domain decomposition method using an FVTD technique for the resolution of an electromagnetic problem on vehicles and evaluated the current on a cable inside the volume of an airplane. Applications of the technique in microwave engineering require both the implementation of electromagnetic sources and the characterization of ports. Baumann [121, 122] introduced new schemes for full-wave field excitation and full-wave S-parameter extraction that make the FVTD method especially well suited for microwave device simulations. To improve the computational efficiency of the FVTD method, Fumeaux [123] introduced a new generalized local time-stepping scheme, which is based on an automatic partition of the computational domain into subdomains where local time steps of the type $2^{l-1}\Delta t$ (l = 1 2 3 . . .) can be applied without violating the stability condition.

E. The Finite Element Time Domain Technique

The finite-element time-domain (FETD or TDFEM) method combines the advantages of a time-domain technique with the versatile spatial discretization options of the finite element method. A variety of FETD methods have been proposed. These schemes generally fall into two categories. Methods in the first category directly discretize the time-dependent Maxwell's equations, yielding an explicit, conditionally stable, time-marching algorithm that can be viewed as a generalization of the finite-difference time-domain (FDTD) method for unstructured grids [124-133]. Methods in the second category discretize the second-order vector wave (curl-curl) equation, obtained by eliminating one of the field variables from Maxwell's equations. The solution of a linear system of equations is required at each time step, but this implicit method can be formulated to be unconditionally stable [134-146].

The explicit FETD has less computational complexity, however the maximum time-step must be constrained to insure stability and it can be relatively difficult to achieve convergence. In implicit FETD methods, the time step is not constrained by a stability criterion and these methods can be extended to higher-orders in a relatively straightforward manner. However, implicit schemes have greater computational complexity because they require a global linear system of equations to be solved at each time step. This can make the simulation of large-scale electromagnetic problems relatively inefficient.

Lou [147] presented a dual-field FETD formulation that computes both the electric and magnetic fields in a leapfrog fashion. This formulation has the advantages of implicit FETD schemes while reducing the computational complexity significantly when the computational domain is split into non-overlapping smaller subdomains. He extended the domain decomposition to the element level in [148].

For unbounded problems, the truncated boundaries of the FETD computational domain need to be properly treated. This is usually done by using conventional absorbing boundary operators [149, 150], boundary integral (BI) methods [151], or PML boundaries [152-160]. As discussed in Section IV, absorbing boundary operators are easy to implement but can exhibit large reflection errors [150]. Boundary integral methods are theoretically exact but computationally expensive. PML implementations yield very small reflection errors in other time-domain methods, such as the FDTD method. However, the FETD formulation of the PML has not been thoroughly investigated.

F. The Time Domain Method of Moments

Most moment method codes solve integral equations in the frequency domain, but it is also possible to use the method of moments to solve time-domain integral equations [161-190]. Consider a perfect electric conductor in free space excited by an incident field $E^i\left(r,t\right)$. This incident field induces a current $J(r,t)$ on the surface S of the conductor that in turn radiates a scattered field. Enforcing the boundary condition on the total magnetic field or electric field on S gives rise to a time domain magnetic field integral equation (TDMFIE) or a time domain electric field integral equation (TDEFIE), respectively.

$$\vec{J}\left(\vec{r},t\right)=2\hat{n}\times\vec{H}^i\left(\vec{r},t\right)+\frac{1}{2\pi}\hat{n}\times\int_s\left[\frac{\vec{J}\left(\vec{r}',t'\right)\times\vec{R}}{R^2}+\frac{1}{Rc}\frac{\partial\vec{J}\left(\vec{r}',t'\right)}{\partial t'}\times\hat{R}\right]ds' \tag{VI-13}$$

$$\frac{4\pi}{\eta}\frac{\partial}{\partial t}\vec{E}^i\left(\vec{r},t\right)\Big|_{Tangent\ to\ \mathrm{S}}=\frac{1}{c}\frac{\partial^2}{\partial t^2}\int_s\frac{\vec{J}\left(\vec{r}',t'\right)}{R}ds'-c\int_s\left(\nabla'\cdot\vec{J}\left(\vec{r}',t'\right)\right)\nabla\left(\frac{1}{R}\right)ds' \tag{VI-14}$$

Note that in the principal value, we essentially exclude the part where the source and observation points are the same (i.e. $R=0$). Since $t'=t-R/c$, and $R\neq 0$, it is always true that $t'<t$. The main difficulty in extending the approach used to solve the frequency domain integral equations comes from the retarded time variable. However, the TDIEs can be solved numerically by means of a marching-on-in-time (MOT) procedure.

Like the method of moments in the frequency domain, the MoM-TD method discretizes the scatterers or targets into segments or patches. The time axis is then divided into equal increments or time steps. The triangular patches and vector basis functions proposed by Rao–Wilton–Glisson (RWG) [8] are commonly used to discretize the current in space and time by expanding the current $J(r,t)$ as a finite linear combination of products of spatial basis functions $S_n(r)$ and temporal basis functions $T_k(t)$

$$J(r,t)=\sum_{j=k}^{N_t}\sum_{n=1}^{N_s} I_{j,n} S_n(r) T_k(t) \tag{VI-15}$$

where the temporal basis functions are generally versions of the same function shifted by a certain number of time steps, $T_k(t)=T(t-k\Delta t)$ with $T(t)=0, \forall\ t<\Delta t$.

To determine the expansion coefficients $I_{j,n}$, Galerkin testing functions are applied in space and point matching is applied at times $t_j = j\Delta t$, $j=1,2,3...$, leading to a set of matrix equations that can be written as,

$$[V]=[Z][I]. \tag{VI-16}$$

The vector [*V*] contains the known incident field quantities and the terms of the Z-matrix are functions of the geometry. The unknown coefficients of the induced current are the terms of the [*I*] vector. These values are obtained by solving the system of equations iteratively. For example, Andriulli [188] proposed an explicit iterative scheme,

$$Z_0 I_j = V_j - \sum_{k=1}^{j} Z_k I_{j-k} \tag{VI-17}$$

where I_j is a $N_s \times 1$ vector, Z_k is a $N_s \times N_s$ matrix relating the currents on the body at time $t_k = k\Delta t$. The current coefficient vector I_j can be obtained once the current coefficient vectors I_k, $k=1,..., j-1$ are known.

The TDIEs have applications and limitations similar to their frequency domain counterparts. The EFIE is suitable for closed and open bodies, while the MFIE is only suitable for smooth, closed bodies. MoM-TD techniques are not very effective when applied to arbitrary configurations with complex geometries or inhomogeneous dielectrics. They also are not well-suited for analyzing the interior of conductive enclosures or thin plates with wire attachments on both sides. However, the time domain MoM is especially well suited for dealing with fast transient electromagnetic fields incident on or radiated from structures in free space.

G. The Generalized Multipole Technique

The *Generalized Multipole Technique* (GMT) was developed in the 1980s by C. Hafner [191] and implemented in software known as the *Multiple Multipole* (MMP) programs. It is essentially a frequency-domain moment-method technique where the basis functions are analytic solutions of the fields generated by sources located some distance away from the surface where the boundary conditions is being enforced. These basis functions are spherical wave field solutions corresponding to multipole sources. By locating these sources away from the boundary, the field solutions form a smooth set of basis functions on the boundary and singularities on the boundary are avoided.

Like the method of moments, a system of linear equations is developed and then solved to determine the coefficients of the basis functions that yield the best solution. Since the basis functions are already field solutions, it is not necessary to do any further computation to determine the fields. Conventional moment methods determine the currents and/or charges on the surface first and then must integrate these quantities over the entire surface to determine the fields. This integration is not necessary at any stage of the GMT solution.

There is little difference in the way dielectric and conducting boundaries are treated by the GMT. The same multipole expansion functions are used. For this reason, a general purpose implementation of the GMT models configurations with multiple dielectrics and conductors much more readily than a general purpose moment method technique.

Despite the advantages of this technique for certain types of modeling, it is not widely used. This may be partly because this technique is a little less intuitive to use and it can be difficult to learn to locate the multipole sources optimally.

Note that the Generalized Multipole Technique should not be confused with the Fast Multipole Method (FMM), which is a technique for exploiting symmetry or periodicity in structures to accelerate some types of electromagnetic modeling codes.

H. The Finite Difference Frequency Domain Technique

Like the finite difference time domain method, the finite difference frequency domain (FDFD) method is based on a finite differential approximation of the derivative operators in the Maxwell curl equations. However, in FDFD, the time-harmonic version of these equations is employed. While time-domain finite difference schemes are very popular, the finite difference frequency domain method has received little attention in the literature. It is essentially similar to the finite element method, but it requires a uniform grid.

VII. Asymptotic-Expansion Based Methods

The techniques described in the previous sections are exact methods in that the error in the numerical solution only comes from the discretization. The numerical solution approaches the exact solution as the discretization is refined. However, as the number of unknowns grows, the demand for computer memory and calculation time also grows. This prohibits these methods from being applied to high frequency problems where the size of the object is much larger than the wavelength. The methods described in this section are based on asymptotic high-frequency expansions of Maxwell's equations. They are high frequency methods that are only accurate when the dimensions of the objects being analyzing are large compared to the wavelength of the field. The asymptotic techniques introduced in the following sections include physical optics, geometrical optics, geometrical theory of diffraction, and uniform theory of diffraction.

A. Physical Optics

The *Physical Optics* (PO) approximation is a well known and efficient method for analyzing large scatters [192]. PO reduces the cost of memory and CPU-time by performing a high frequency approximation. It is a current-based method in which the physical optics approximation is used to obtain the current density induced on a surface. The surface current density, $\mathbf{J_s}$, can be determined by,

$$\mathbf{J_S} = \hat{n} \times \left(\mathbf{H^i} + \mathbf{H^r} \right) \tag{VII-1}$$

where $\mathbf{H^i}$ and $\mathbf{H^r}$ represent the incident and reflected magnetic field components evaluated on the surface. $\hat{n}$ is the unit vector normal to the surface. If the surface can be approximated as an infinite plane surface, then by image theory,

$$\hat{n} \times \mathbf{H^i} = \hat{n} \times \mathbf{H^r} \tag{VII-2}$$

and Equation (VII-1) reduces to

$$\mathbf{J_S} = \hat{n} \times \left(\mathbf{H^i} + \mathbf{H^r} \right) = 2\hat{n} \times \mathbf{H^i} = 2\hat{n} \times \mathbf{H^r} \tag{VII-3}$$

The electric and magnetic field radiated by the surface current on the illuminated side of the reflector can be determined by [194],

$$\mathbf{E}(r) = \frac{j}{w\varepsilon_0} \nabla \times \nabla \times \int_{S_r} J_S\left(r, r'\right) g(r, r')\, dS' \tag{VII-4}$$

$$\mathbf{H}(r) = \nabla \times \int_{S_r} J_S\left(r, r'\right) g(r, r')\, dS' \tag{VII-5}$$

where $g(r, r') = \dfrac{e^{jk|r-r'|}}{|r-r'|}$.

Equation (VII-3) is exact only when the surface is infinitely large. The accuracy of the approximation depends on the transverse dimensions of the reflecting surface, the radius of curvature, location of edges, and the angle of the incident field. Generally, PO works well for large, smooth surfaces with low curvature. The implicit assumption for the physical optics approximation is that the incident field is treated as a locally planar wave. Also, it assumes that the reflector surface is perfectly conducting.

It has been found that PO provides an accurate prediction of far-field patterns of reflected antennas in the main beam region and out to several side lobes [196]. The major disadvantage of PO is that the integration over the surface of the reflector may be quite complicated and time consuming when the feed is placed off-axis or the feed pattern is asymmetric [197]. Moreover, the radiation integral has to be evaluated each time the observation point is changed.

Fast and efficient evaluation of the radiation integral was proposed using a fast series approach [196], incorporating a multilevel fast multipole method [197], or decomposing the scatterer into subdomains [198]. Initially applied in the frequency domain, PO has also been extended into the time domain [199].

B. Geometrical Optics (Ray Optics)

Geometrical Optics (GO) [202] or geometrical optics with aperture integration (GO/AI) is a ray-based method intended for the consideration of electrically large dielectric structures in applications like the analysis of reflector antennas. In GO analysis, geometrical optics techniques (ray tracing) are used to set up equivalent currents on an aperture plane which is normal to the axis of the reflector. Then, the tangential aperture fields are constructed and used to determine the radiated fields utilizing the Fourier transform. Different formulations are obtained based on the use of aperture electric fields, magnetic fields or their combinations [201]. The advantage of the GO/AI method is that the integration over the aperture plane can be performed with equal ease for any feed pattern or feed position [195].

The relationship between GO and PO was demonstrated in [201]. It was shown that the PO integral can be represented as a summation of many Fourier transforms, such that the first few terms resemble the GO representation. Using the “extinction theorem” [194], the fields predicted by the integration of PO surface currents were shown to agree with the geometrical optics aperture fields on the aperture plane to within the local plane wave approximation. It was concluded that the accuracy obtained by the two methods is comparable.

C. Geometrical Theory of Diffraction

The approximations in both physical optics and geometrical optics are based on the following assumptions [194]:

- The current density is zero on the shadow side of the reflector
- The discontinuity of the current density over the rim of the reflector is neglected
- Direct radiation from the feed and aperture blockage by the feed are neglected.

Both PO and GO ignore the edge diffractions which are highly dependent on the whether the edges of the reflector are flared, sharp, absorber lined or serrated. Thus, they cannot accurately predict the far fields beyond the first few side lobes. For predicting the patterns more accurately in all regions, geometrical diffraction techniques are required.

As an extension of GO, the *Geometrical Theory of Diffraction* (GTD) overcomes the limitations of GO by introducing a diffraction mechanism [203]. The diffracted field is determined at the points on the surface where there is a discontinuity in the incident and reflected field. The value of the diffracted field is evaluated at these points with the aid of an appropriate diffraction coefficient. Usually, the coefficient is determined from asymptotic solutions of simple boundary-value problems with so called canonical geometries, such as a conducting wedge, cylinder or sphere. Since the solutions of these canonical problems are known, the object under investigation can be partitioned into smaller components, so that each component represents a canonical geometry. The ultimate solution is a superposition of the contributions from each component [193].

Two major advantages of GTD over other high frequency asymptotic techniques are that it provides insight into the radiation and scattering mechanisms from the various parts of the structure, and it can yield more accurate results. The method has attracted increasing attention; especially for applications to reflector antennas [204-209]. Unfortunately, GTD fails in the transition region adjacent to the shadow boundary, at caustics (points through which all the rays of a wave pass), or in close proximity to the surface of the scatterer. In these zones, the field cannot be treated as a plane wave. Thus, ray techniques become invalid. To deal with this problem, a number of alternative approaches have been proposed: uniform solutions [211-212], methods for dealing with caustic curves [213-215], physical theory of diffraction (PTD) [216], and the spectral theory of diffraction (STD) [217-218]. A comprehensive introduction to these methods can be found in [210].

D. Uniform Theory of Diffraction

The *Uniform Theory of Diffraction* (UTD) is a uniform version of the geometrical theory of diffraction. It was initially proposed [219] to deal with the problem that GTD produces inaccurate results at the shadow boundaries. The uniform theory of diffraction approximates near electromagnetic fields as quasi-optical and uses ray diffraction to determine diffraction coefficients for each diffracting object-source combination. These coefficients are then used to calculate the field strength and phase for each direction away from the diffracting point.

VIII. Hybrid Methods

Many practical problems are too complicated to be solved accurately by a single numerical or asymptotic method. It is often advantageous to combine two numerical modeling techniques in a single field solver in order to take advantage of the strengths of each technique to solve problems that neither technique alone could model efficiently. For example, a finite element method can be combined with a boundary element method to form a more powerful hybrid numerical technique for analyzing both open-region problems and complex inhomogeneous objects. A full-wave numerical technique can also be combined with an asymptotic method to model very large objects with small features that require detailed analysis, such as an antenna mounted on an airplane.

Hybrid methods are not simply two separate modeling codes with a common user interface. A hybrid method generally divides a problem into two parts and applies a different technique to each part while matching the currents or fields at the boundary to ensure a unique solution. Some hybrid codes solve one part first, and then use the boundary fields as the sources when solving the second part. Other hybrid codes solve both parts simultaneously allowing the solution of each part to influence the solution of the other.

Formulations for many hybrid techniques have been developed and reported in the literature. Some of the more common (and useful) hybrid techniques are described in the following sections.

A. Hybrid FEM/BEM

Finite element methods excel at modeling complex volumetric structures, but are weak when it comes to modeling thin wires and unbounded radiation problems. Boundary element methods excel at modeling wires and unbounded geometries, but do not model complex structures that include a variety of materials well. The complementary strengths of these two methods make them ideal candidates for hybridization. Full-wave FEM/BEM (also called FEM/MOM, FE-BE and FE-BI) techniques have been successfully used to model many problems that could not be modeled effectively using either of the two techniques alone [220-227].

Hybrid FEM/BEM techniques introduce a fictitious surface (which may or may not coincide with an actual material surface) that separates an interior volume from an exterior volume. The interior region is analyzed using a finite element method with unknown electric or magnetic surface currents establishing the boundary condition on the outer surface. The exterior region is analyzed using a boundary element method, with unknown electric or magnetic currents on the fictitious surface. Two sets of matrix equations are developed that share unknowns on the boundary between the interior and exterior volumes. By forcing the fields on both sides of the fictitious surface to be consistent with each other, the two matrix equations can be combined into one larger equation with a unique solution.

In practice, it is relatively inefficient to generate one large matrix that is partly dense and partly sparse. *Inward looking* techniques repeatedly solve the finite element portion of the problem, while populating the boundary element method matrix. *Outward looking* techniques repeatedly solve the boundary element method portion while populating the finite element matrix. Choosing the right hybrid technique for a particular application greatly increases the efficiency of a hybrid FEM/BEM approach [228].

B. Hybrid MOM/GTD, MOM/PO

Asymptotic methods can deal with objects whose overall dimensions are large in terms of the wavelength. However, if large objects contain features that are too fine to be analyzed by an asymptotic method, it becomes necessary to employ hybrid methods that combine asymptotic techniques with numerically rigorous methods. Techniques that combine a moment method with an asymptotic method can be broadly categorized as either ray-based or current-based. Ray-based methods, such as MOM/GTD, provide a considerable speed advantage, but can be difficult to implement for arbitrary and complex objects [237].

Hybrid MOM/GTD techniques were first described in the 1970s [229-230]. Since GTD fails in regions where the field cannot be approximated by a local plane wave, uniform solutions have been developed that overcome some of the limitations of GTD. Hybrid approaches combining MOM and UTD are discussed in [231-233].

Physical optics is a current-based asymptotic method. The hybridization of PO with MOM has two advantages over the combination of ray-based methods and MOM. First, since both MOM and PO are current-based, they can be easily blended on the same surface. Second, MOM/PO is relatively general in that there are no specific restrictions on the geometries that can be modeled [234]. Consequently, this hybrid technique has received considerable attention in the literature [234-241]. It has also been implemented in a commercial numerical code [242].

C. Hybrid FEM/PO

Hybrid methods that combine high-frequency asymptotic techniques with the method of moments are not suitable for solving problems with inhomogeneous or anisotropic materials. These types of problems are better suited for analysis by a hybrid FEM/PO technique. The hybridization of FEM and asymptotic techniques is described in [243-244].

IX. COMMERCIAL CEM SOFTWARE SURVEY

As part of this project, lists of free and commercial CEM modeling codes were developed and published on the web at the following URLs,

Free CEM Codes: http://www.cvel.clemson.edu/modeling/EMAG/free-codes.html

Commercial CEM Codes: http://www.cvel.clemson.edu/modeling/EMAG/csoft.html.

The companies or contacts associated with each code in these lists were asked to complete a brief online survey describing the general features, capabilities and costs of their products. Their responses are summarized in the following tables. In some cases, the responses received on the survey forms were inconsistent with the information on the company's web site or with the first hand experience of the authors. We've done our best to insure that the information provided in the tables below is accurate. Any errors or omissions can be reported to CVEL-L@clemson.edu. Reported corrections will be made to the information posted on the web site.

A. 3D Full-Wave Codes

The main emphasis of this survey was on general purpose 3D full-wave codes. Twelve of the codes included in the survey responses were placed in this category. Note that some of the codes in the "hybrid" category can also be used as general purpose full-wave codes.

Table 8: General Purpose 3D Full-Wave Codes

Software Name	Company	Technique	Cost	Comment
AMDS Antenna Modeling Design System	Agilent	FDTD	between \$10,000 and \$50,000	for wireless appliance design and modeling
ApsimFDTD	Applied Simulation Technology	FDTD	between \$10,000 and \$50,000	part of a suite of tools for IC and package analysis
CST Microwave Studio – Transient Solver	Computer Simulation Technology	FDTD	between \$10,000 and \$50,000	part of a suite of tools
CST Microwave Studio – Frequency Domain Solver	Computer Simulation Technology	FEM	between \$10,000 and \$50,000	part of a suite of tools
Comsol Multiphysics – RF Module	Comsol	FEM	between \$10,000 and \$50,000	part of a suite of tools

Efield FD-Startpack	Efield AB	BEM	between $10,000 and $50,000	part of a suite of tools
Efield TD-Startpack	Efield AB	FDTD	between $10,000 and $50,000	part of a suite of tools
EMA3D	Electro Magnetic Applications	FDTD	between $10,000 and $50,000	
EMDS	Agilent Technologies	FEM	between $10,000 and $50,000	EMDS was integrated into Agilent ADS in Aug. 2008.
emGine Environment	Petr Lorenz	TLM	between $10,000 and $50,000	GUI is open source. Binaries are free for non-commercial use.
EMPIRE XCcel	Empire	FDTD	between $10,000 and $50,000	
EZ-EMC	EMS-Plus	FDTD	between $1000 and $10,000	
EZ-FDTD	EMS-Plus	FDTD	between $1000 and $10,000	
EZNEC Pro	EZNEC	BEM	between $200 and $1000	wire and wire-grid modeling
Fidelity	Zeland Software	FDTD	not reported	did not respond to survey
GEMS	Computer and Communication Unlimited	FDTD	between $10,000 and $50,000	
HFSS	Ansys/Ansoft	FEM	greater than $50,000	
IE3D	Zeland Software	BEM	not reported	Did not respond to survey

LC	LC	FDTD	FREE	
MaX-1	John Wiley & Sons	GMT	between $1000 and $10,000	package includes 2D and 3D FDTD solvers
MEEP	MIT	FDTD	FREE	Free software under the GNU GPL.
MEFiSTo-3D Pro	Faustus Scientific Corporation	TLM	between $1000 and $10,000	numerical TLM simulation engines for 2D problems are also included.
NEC2	Lawrence Livermore Lab	BEM	FREE	wire grid modeling plus simple plates modeled by point matching, text file input and output
NEC4	Lawrence Livermore Lab	BEM	between $200 and $1000	U.S. export controlled
PAM-CEM	ESI Group	FDTD	between $10,000 and $50,000	
PhysPack	Physware	BEM	greater than $50,000	chip-package-board simulation
SEMCAD X	Schmid & Partner Engineering	FDTD	between $10,000 and $50,000	package includes quasi-static solvers
Toy	The CEMTACH Group	FDTD	FREE	for educational use
ToyTLM	The CEMTACH Group	TLM	FREE	for educational use
WIPL-D Pro	WIPL-D	BEM	between $10,000 and $50,000	metallic and dielectric 3D structures with wires and plates
XFDTD	Remcom	FDTD	between $10,000 and $50,000	

B. 3D Quasi-Static Codes

In many situations, it is better to use a quasi-static modeling code to model components that are small relative to the wavelengths of interest, even when these components are used at RF or microwave frequencies. Generally, quasi-static modeling codes are more powerful and more efficient for modeling complex electrically small geometries than full-wave codes. The codes listed in the Table 9 are electrostatic and/or magnetostatic modeling codes.

Table 9: 3D Quasi-Static Codes

Software Name	Company	Technique	Cost	Comment
AMaze - HiPhi	Field Precision	FEM	between $1000 and $10,000	electrostatic (part of a suite of tools)
AMaze - Magnum	Field Precision	FEM	between $1000 and $10,000	magnetostatic (part of a suite of tools)
Amperes	Integrated Engineering Software	BEM	between $10,000 and $50,000	magnetostatic
Coulomb	Integrated Engineering Software	BEM	between $10,000 and $50,000	electrostatic
Comsol Multiphysics – AC/DC module	Comsol	FEM	between $10,000 and $50,000	electrostatic, magnetostatic suite of tools
EMS	ElectromagneticWorks	FEM	between $1000 and $10,000	electrostatic, magnetostatic suite of tools
EMPLab	EM Photonics	FDTD	between $1000 and $10,000	MATLAB based software
Faraday	Integrated Engineering Software	BEM	between $10,000 and $50,000	magnetostatic
Flux3D	Magsoft	FEM	between $1000 and $10,000	electrostatic and magnetostatic tools
Opera 3D	Vector Fields Inc.	FEM	between $1000 and $10,000	electrostatic and magnetostatic tools

MagNet	Infolytica Corporation	FEM	between $10,000 and $50,000	magnetostatic 2D/3D
Maxwell	Ansys/Ansoft	FEM	not reported	electrostatic / magnetostatic (includes 2D tool)
ElecNet	Infolytica Corporation	FEM	between $10,000 and $50,000	electrostatic 2D/3D
PCBMod	Simlab GmbH	2D BEM or 3D PEEC	between $10,000 and $50,000	electrostatic
Q3D Extractor	Ansys/Ansoft	FEM	greater than $50,000	electrostatic and magnetostatic tools

C. 2D High-Frequency Codes

Great insight into wave propagation, reflection, diffraction, shielding, etc. can be obtained from 2D high-frequency modeling codes. 2D codes use a fraction of the computational resources required by 3D codes; and it is much simpler to generate 2D inputs and visualize 2D outputs.

Table 10: 2D High-Frequency Codes

Software Name	Company	Technique	Cost	Comment
MEFiSTo-3D Pro	Faustus Scientific Corporation	TLM	between $1000 and $10,000	2D simulation engines packaged with 3D tool
Momentum	Agilent	BEM	between $10,000 and $50,000	2.5D code for planar circuits
TriComp - WaveSim	Field Precision	FEM	between $1000 and $10,000	part of a suite of tools

D. 2D Quasi-Static Codes

2D quasi-static codes are excellent tools for visualizing electric and magnetic field distributions. Generally, these tools have a relatively low cost and do not require a lot of computer resources.

Table 11: 2D Quasi-Static Codes

Software Name	Company	Technique	Cost	Comment
TriComp - EStat	Field Precision	FEM	between \$200 and \$1000	electrostatic
TriComp - PerMag	Field Precision	FEM	between \$200 and \$1000	magnetostatic
Finite Element Method Magnetics (FEMM)	Dr. David Meeker	FEM	FREE	electrostatic / magnetostatic
Electro	Integrated Engineering Software	BEM	between \$10,000 and \$50,000	electrostatic
Flux2D	Magsoft	FEM	between \$1000 and \$10,000	electrostatic, magnetostatic suite of tools
Magneto	Integrated Engineering Software	FEM / BEM	between \$10,000 and \$50,000	includes BEM and FEM solvers
Maxwell	Ansys/Ansoft	FEM	not reported	electrostatic / magnetostatic (includes 3D tool)
Oersted	Integrated Engineering Software	FEM / BEM	between \$10,000 and \$50,000	includes BEM and FEM solvers
pdnmesh		FEM	FREE	electrostatic
Q2D	Ansys/Ansoft	FEM	greater than \$50,000	bundled with Q3D Extractor
QuickField	Tera Analysis	FEM	not reported	did not respond to survey

E. Hybrid Codes

Hybrid codes combine the features of two different modeling techniques in order to be able to model a wider range of problem geometries. Hybrid codes are not suites of tools that share a common user interface. Hybrid codes are capable of simultaneously applying two different solvers to different regions of a problem geometry. Each of the codes below is also capable of employing just a single technique and therefore could have been included with the 3D full-wave codes listed in Table 8.

Table 12: Hybrid Codes

Software Name	Company	Technique	Cost	Comment
CONCEPT-II	Technical University of Hamburg-Harburg	Hybrid BEM and PO	between \$1000 and \$10,000	large wire and thin plate structures
GEMACS	Applied Research Associates	BEM-FDFD-UTD	between \$200 and \$1000	large wire and thin plate structures, with dielectric capability
FEKO	EM Software and Systems	BEM-FEM-PO	between \$10,000 and \$50,000	
Singula	Integrated Engineering Software	BEM-PO	between \$10,000 and \$50,000	
SuperNEC	Poynting Antennas	BEM -PO	between \$1000 and \$10,000	
efield	Efield AB	hybrid FDTD-FEM-TD hybrid MoM-MLFMM-PO	greater than \$50,000	two different hybrid tools
EMAP5	Clemson University	BEM-FEM	FREE	

F. Special Purpose Codes

The codes below employ numerical electromagnetic modeling techniques, but are optimized for specific applications.

Table 13: Special Purpose Codes

Software Name	Company	Technique	Cost	Comment
AMaze - OmniTrak	Field Precision	FEM	between $1000 and $10,000	3D charged particle beam analysis
Analyst	Simulation Technology & Applied Research	FEM-TD, FEM	between $10,000 and $50,000	suite of tools for designing microwave and accelerator components
Antenna Model	Teri Software	BEM	less than $200	antenna model uses the MININEC code, used for the analysis of wire antennas
AXIEM	AWR	BEM	between $10,000 and $50,000	for modeling planar circuits
CableMod	Simlab GmbH	BEM	between $10,000 and $50,000	2D electrostatic solver for calculating transmission line parameters and crosstalk
Compliance	Quantic EMC	BEM	between $10,000 and $50,000	calculates currents on circuit traces (2D BEM), then calculates radiated fields in 3D
EM Explorer	EM Explorer	FDTD	between $1000 and $10,000	solver for scattering problems of periodic structures
EM3DS	MEM Research	BEM	between $1000 and $10,000	MMIC modeling
EMFlex	Weidingler Associates Inc.	FEM-TD	between $10,000 and $50,000	optical modeling
EZ-PowerPlane	EMS-Plus	Cavity Resonance	between $1000 and $10,000	modeling power bus noise in printed circuit boards
FlexPDE	PDE Solutions	FEM-TD	between $1000 and $10,000	general PDE solver - can be 1D, 2D or 3D

GSolver	Grating Solver Development Company		between $1000 and $10,000	periodic (grating) boundaries
HFWorks	ElectromagneticWorks	FEM	between $10,000 and $50,000	RF, Microwave and MMwave component analysis
JCMsuite	JCMwave GmbH	FEM	between $10,000 and $50,000	For optical modeling
Open FMM	Open FMM	BEM	FREE	Large scattering problems
OptEM Inspector	OptEM Engineering	not reported	between $10,000 and $50,000	RC extraction software tool for deep submicron (DSM) IC designs
OptEM Cable Designer	OptEM Engineering	not reported	between $10,000 and $50,000	For modeling multi-conductor flex, unshielded twisted-pair and twisted-pair cables
ScatLab	ScatLab	Mie theory, T-Matrix method	FREE	Large Scattering Problems
Speed2000	Sigrity	FDTD	not reported	for modeling interactions in multi-layer chip packages and printed circuit boards
Trace Analyzer	Trace Analyzer	BEM or MOM	between $200 and $1000	Electrostatic solution to calculate the [C] matrix, and the R/L/G matrices are derived from [C]
TriComp - Trak	Field Precision	FEM	between $1000 and $10,000	2D charged particle beam analysis
XGtd	Remcom	UTD/GTD	between $10,000 and $50,000	3D high-frequency ray tracing code.

REFERENCES

[1] Maxwell, James Clerk, "A Dynamical Theory of the Electromagnetic Field", *Philosophical Transactions of the Royal Society of London 155*, pp. 459-512, 1865. (This article accompanied a December 8, 1864 presentation by Maxwell to the Royal Society.)

[2] K. S. Yee, “Numerical solution of initial boundary value problems involving Maxwell’s equations in isotropic media,” *IEEE Trans. Antennas Propagat.*, vol. 14, no. 4, pp. 302-307, 1966.

[3] R. F. Harrington, “Matrix Methods for Field Problems,” *Proc. IEEE*, vol. 55, no. 2, pp. 136-149, Feb. 1967.

[4] T. Hubing, “Survey of Numerical Electromagnetic Modeling Techniques,” University of Missouri-Rolla, Department of Electrical Engineering, *TR91-1-001.3*, Sep. 1991.

[5] R. F. Harrington, *Field Computation by Moment Methods*, Macmillan Publishing Company, New York, 1968. (reprinted by IEEE Press in 1993.)

[6] R. F. Harrington, *Time Harmonic Electromagnetic Fields*, New York, McGraw Hill, 1961, Ch. 3.

[7] R. E. Collin, *Field Theory of Guided Waves, 2nd ed.*, New York, IEEE Press, 1991, Ch. 2.

[8] S. M. Rao, D. R. Wilton, and A. W. Glisson, “Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, vol. 30, pp. 409–418, May 1982.

[9] D. C. Chang and J. X. Zheng, “Electromagnetic modeling of passive circuit elements in MMIC,” *IEEE Trans. on Microwave Theory and Tech.*, vol. 40, no. 9, pp. 1741 – 1747, Sep. 1992.

[10] P. Silvester, “High-order finite element waveguide analysis (Program Descriptions),” *IEEE Trans. on Microwave Theory and Tech.*, vol. 17, no. 8, pp. 651-652, Aug. 1969.

[11] O. C. Zienkiewicz, A. K. Bahrani, and P. L Arlett, “Numerical solution of 3-dimensional field problems,” *Proc. IEE* (London), vol. 115, pp. 367-369, Feb. 1968.

[12] B. H. McDonald and A. Wexler, “Finite-element solution of unbounded field problems,” *IEEE Trans. on Microwave Theory and Tech.*, vol. 20, no. 12, pp. 841-847, Dec. 1972.

[13] D. R. Lynch and K. D. Paulsen, “Origin of Vector Parasites in Numerical Maxwell Solutions,” IEEE Trans. on Microwave Theory and Techniques, vol. 39, no. 3, pp. 383-394, Mar. 1991.

[14] D. R. Lynch and K. D. Paulsen, “Elimination of Vector Parasites in Numerical Maxwell Solutions,” IEEE Trans. on Microwave Theory and Techniques, vol. 39, no. 3, pp. 395-404, Mar. 1991.

[15] Y. Li and Z. Cendes, “High-accuracy absorbing boundary condition,” IEEE Trans. on Magnetics, vol. 31, no. 3, pp. 1524-1529, May 1995.

[16] A. Bayliss, E. Turkel, "Radiation boundary conditions for wave-like equations," *Communications on Pure and Applied Mathematics*, vol. 33, pp. 707--725, 1980.

[17] A. Bayliss, M. Gunzburger, E. Turkel, "Boundary conditions for the numerical solution of elliptic equations in exterior regions," *SIAM Journal of Applied Mathematics*, vol. 42, pp. 430-451, 1982.

[18] T.G. Moore, J.G. Blaschak, A. Taflove, G.A. Kriegsmann, "Theory and application of radiation boundary operators," *IEEE Trans. Antennas Propagat.*, vol. 36, pp. 1797-1812, 1988.

[19] J. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comput. Phys.* 114, pp. 185-200, 1994.

[20] J. M. Jin, and W. C. Chew, "Combining PML and ABC for finite element analysis of scattering problems," *Micro. Opt. Tech. Lett*., vol. 12, pp. 192—197, 1996.

[21] Wu J-Y, Kingsland DM, Lee J-F, Lee R. , "A Comparison of anisotropic PML to Berenger's PML and its application to the finite-element method for EM scattering," *IEEE Trans. on Antennas and Propagat*., vol. 45, no.1, pp. 40-50, 1997.

[22] R. Mittra and M. Kuzuoglu, "A review of some recent advances in perfectly-matched absorbers for mesh truncation in FEM," *IEEE Antennas and Propagation Society International Symposium*, 1997 Digest, vol. 2, pp.1302 -1305, 1997.

[23] M. Ali, T. H. Hubing, and J. Drewniak, "A hybrid FEM/MOM technique for electromagnetic scattering and radiation from dielectric objects with attached wires," *IEEE Trans. on Electromag. Compat*., vol. 39, no. 4, pp. 304-314, Nov. 1997.

[24] B. Engquist and A. Majda, "Absorbing boundary conditions for the numerical simulation of waves," *Mathematics of Computation*, 31, pp. 629-651, 1977.

[25] E. L. Lindman, "'Free-space' boundary conditions for the time dependent wave equation," *Journal of Computational Physics*, 18, pp. 67-78, 1975.

[26] G. Mur, "Absorbing boundary conditions for the finite-difference approximation of the time-domain electromagnetic-field equations," *IEEE Trans. on Electromag. Compat.,* vol. 23, no. 4, pp. 377-382, Nov. 1981.

[27] Z. P. Liao, H. L. Wong, B.-P. Yang, and Y.-F. Yuan, "A transmitting boundary for transient wave analysis," *Science Sinica*, Series A, 27, 10, pp. 1063-1076, 1984.

[28] R. G. Keys, "Absorbing boundary conditions for acoustic media," *Geophysics*, 50, 6, pp. 892-902, 1985.

[29] R. L. Higdon, "Numerical absorbing boundary conditions for the wave equation," *Mathematics of Computation*, 49, 179, pp. 65-90, 1987.

[30] C. Rappaport and L. Bahrmasel, "An absorbing boundary condition based on anechoic absorber for EM scattering computation," *Journal of Electromagnetic Waves and Applications*, 6, 12, pp. 1621-1634, 1992.

[31] J. P. Berenger, "A perfectly matched layer for the absorption of electromagnetics waves," *Journal of Computational Physics*, 1 14, 1, pp. 185-200, 1994.

[32] W. Andrew, C. Balanis, and P. Tirkas, “A comparison of the Berenger perfectly matched layer and the Lindman higher-order ABCs for the FDTD method,” *IEEE Microwave and Guided Wave Letters*, 5, 6, pp. 192-194, 1995.

[33] C. J. Railton and E. M. Daniel, “Comparison of the effect of discretisation on absorbing boundary algorithms in finite difference time domain method,” *Electronics Letters*, 28, 20, pp. 1891-1893, 1992.

[34] V. Betz and R. Mittra, “Comparison and evaluation of boundary conditions for the absorption of guided waves in an FDTD simulation,” *IEEE Microwave and Guided Wave Letters*, 2, 12, pp. 499-501, 1992.

[35] R. Diaz and I. Scherbatko, “A new radiation boundary condition for FDTD based on self-teleportation of fields,” *2003 IEEE Microwave Symposium Digest*, 3, 8, pp. 2073-2076, 2003.

[36] A.C. Cangellaris and D.B. Wright, “Analysis of the numerical error caused by the stair-stepped approximation of a conducting boundary in FDTD simulations of electromagnetic phenomena,” *IEEE Trans. Antennas Propagat.*, vol. 39, no. 10, pp. 1518-1525, 1991.

[37] R. Holland, “Pitfalls of staircase meshing,” *IEEE Trans. on Electromag. Compat.*, vol. 35, no. 4, pp. 434-439, Nov. 1993.

[38] K. K. Mei, A. Cangellaris, and D. J. Angelakos, “Conformal time domain finite difference method,” *Radio Science*, 19, 5, pp. 1145-1 147, 1984.

[39] K. S. Yee, J. S. Chen, and A. H. Chang, “Conformal finite difference time-domain (FDTD) with overlapping grids,” *IEEE Trans. Antennas Propagat.*, AP-40, 9, pp. 1068- 1075, 1992.

[40] T. G. Jurgens, A. Taflove, K. Umashankar, and T. G. Moore, “Finite-difference time-domain modeling of curved surfaces,” *IEEE Trans. Antennas Propagat.*, AP-40, 4, pp. 357-366, 1992.

[41] T. G. Jurgens and A. Taflove, “Three-dimensional contour FDTD modeling of scattering from single and multiple bodies,” *IEEE Trans. Antennas Propagat.*, AP-41, 12, pp. 1703-1708, 1993.

[42] S. Dey and R. Mittra, “A locally conformal finite-difference time-domain (FDTD) algorithm for modeling three-dimensional perfectly conducting objects,” *IEEE Microwave and Guided Wave Letters*, vol. 7, no. 9, pp. 273–275, Sep. 1997.

[43] W. Yu and R. Mittra, “A Conformal Finite Difference Time Domain Technique for Modeling Curved Dielectric Surfaces,” *IEEE Microwave and Guided Wave Letters*, vol. 11, no. 1, Jan. 2001.

[44] I. A. Zagorodnov, R. Schuhmann, and T.Weiland, “A uniformly stable conformal FDTD-method in cartesian grids,” *Int. J. Numer. Modelling- Electron. Networks Devices Fields*, vol. 16, pp. 127–141, 2003.

[45] W. Sha, X. Wu, Z. Huang, and M. Chen, “ A new FDTD (2,4) scheme for modeling three-dimensional curved perfectly conducting objects,” *IEEE Microwave and Wireless Components Letters*, 18, 3, pp. 149-151, 2008.

[46] R. Holland and L. Simpson, “Finite-difference analysis EMP coupling to thin struts and wires,’’ *IEEE Trans. on Electromag. Compat.*, vol. 23,no. 2, pp. 88-97, 1981.

[47] K. S. Yee, “Numerical solution to Maxwell’s equations with non-orthogonal grids,” Tech. Rep. UCRL-93268, Lawrence Livermore National Laboratory, 1987.

[48] A. Taflove, K. R. Umashankar, B. Beker, F. Harfoush, and K. S. Yee, “Detailed FD-TD analysis of electromagnetic fields penetrating narrow slots and lapped joints in thick conducting screens,” *IEEE Trans. Antennas Propagat.*, vol. 36, no. 2, pp. 47-257, 1988.

[49] K. R. Demarest, “A finite difference-time domain technique for modeling narrow apertures in conducting scatterers,” *IEEE Trans. Antennas Propagat.*, vol. 35, no. 7, pp. 826-831, 1987.

[50] C. D. Turner and L. D. Bacon, “Evaluation of a thin-slot formalism for finite-difference time-domain electromagnetics codes,” *IEEE Trans. on Electromag. Compat.*, vol. 30, no. 4, pp. 523-528, 1988.

[51] D. J. Riley and C. D. Turner, “The inclusion of wall loss in finite-difference time-domain thin-slot algorithms,” *IEEE Trans. on Electromag. Compat.*, vol. 33, no. 4, pp. 304-311, 1991.

[52] J. H. Oates and R. T. Shin, “Small aperture modeling for EM1 applications using the finite-difference time-domain technique,” *Journal of Electromagnetic Waves and Applications*, 9, 112, pp. 37-69. 1995.

[53] B.-Z. Wang, “Enhanced thin-slot formalism for the FDTD analysis of thin-slot penetration,” *IEEE Microwave and Guided Wave Letters*, 5, 5, pp. 142-143, 1995.

[54] D. Hockanson, J. Drewniak, T. Hubing and T. Van Doren, “FDTD modeling of common-mode radiation from cables,” *IEEE Trans. on Electromag. Compat.*, vol. 38, no. 3, pp. 376-387, Aug. 1996.

[55] K.-P. Ma, M. Li, J. Drewniak, T. Hubing and T. Van Doren, “Comparison of FDTD algorithms for subcellular modeling of slots in shielding enclosures,” *IEEE Trans. on Electromag. Compat.*, vol. 39, no. 2, pp. 147-155, May 1997.

[56] R. J. Luebbers, F. Hunsberger, K. S. Kunz, R. B. Standler, and M. Schneider, “A frequency-dependent finite-difference time-domain formulation for dispersive materials,” *IEEE Trans. on Electromag. Compat.*, vol. 32, no. 3, pp. 22-221, Aug. 1990.

[57] T. Kashiwa, N. Yoshida, and I. Fukai, “A treatment by the finite-difference time-domain method of the dispersive characteristics associated with orientation polarization,” *Transactions IEICE*, E73, 8, pp. 1326-1328, 1990.

[58] T. Kashiwa and I. Fukai, “A treatment by the FD-TD method of the dispersive characteristics associated with electronic polarization,” *Microw. Opt. Technol. Lett.,*, 3, 6, pp. 203- 205, 1990.

[59] T. Kashiwa, Y. Ohtomo, and I. Fukai, “A finite-difference time-domain formulation for transient propagation in dispersive media associated with Cole-Cole’s circular ARC law,” *Microw. Opt. Technol. Lett.*, 3, 12, pp. 416-419, 1990.

[60] R. Pontalti, L. Cristoforetti, R. Antolini, and L. Cescatti, "A multi-relaxation $(FD)^2$-TD method for modeling dispersion in biological tissues," *IEEE Trans. on Microwave Theory Techniques*, vol. 42, no. 3, pp. 526-527, 1994.

[61] P. G. Petropoulos, "Stability and phase error analysis of FDTD in dispersive dielectrics," *IEEE Trans. Antennas Propagat.,* vol. 42, no. 1, pp. 62-69, 1994.

[62] J. G. Maloney and G. S. Smith, "The use of surface impedance concepts in the finite-difference time-domain method," *IEEE Trans. Antennas Propagat.*, vol. 40, no. 1, pp. 38-48, 1992.

[63] K. S. Yee, K. Shlager, and A. H. Chang, "An algorithm to implement a surface impedance boundary condition for FDTD," *IEEE Trans. Antennas Propagat.,* vol. 40, no. 7, pp. 833-837, 1992.

[64] S. Kellali, B. Jecko, and A. Reineix, "Implementation of a surface impedance formalism at oblique incidence in FDTD method," *IEEE Trans. on Electromag. Compat.,* vol. 35, no. 3, pp. 347-356, 1993.

[65] C. J. Railton and J. P. McGeehan, "An analysis of microstrip with rectangular and trapezoidal conductor cross sections," *IEEE Trans. on Microwave Theory Techniques*, vol. 38, no. 8, pp. 1017-1022, 1990.

[66] J. G. Maloney and G. S. Smith, "The efficient modeling of thin material sheets in the finite-difference time-domain (FDTD) method," *IEEE Trans. Antennas Propagat.,* vol. 40, no. 3, pp. 323-330, 1990.

[67] J. G. Maloney and G. S. Smith, "A comparison of methods for modeling electrically thin dielectric and conducting sheets in the finite-difference time-domain (FDTD) method," *IEEE Trans. Antennas Propagat.,* vol. 41, no. 5, pp. 690-694, 1993.

[68] A. Taflove, "Review of the formulation and applications of the finite-difference time-domain method for numerical modeling of electromagnetic wave interactions with arbitrary structures," *Wave Motion*, 10, 6, pp. 547-582, 1988.

[69] I. S. Kim and W. J. R. Hoefer, "Numerical dispersion characteristics and stability factor for the TD-FD method," *Electronics Letters,* 26, 7, pp. 485-487, 1990.

[70] S. L. Ray, "Numerical dispersion and stability characteristics of time-domain methods on nonorthogonal meshes," *IEEE Trans. Antennas Propagat.,* vol. 41, no. 2, pp. 233-235, 1993.

[71] K. L. Shlager, J. G. Maloney, S. L. Ray, and A. F. Peterson, "Relative accuracy of several finite-difference time-domain methods in two and three dimensions," *IEEE Trans. Antennas Propagat.,* vol. 41, no. 12, pp. 1732-1737, 1993.

[72] A. Zhao, "Analysis of the numerical dispersion of the 2D alternating-direction implicit FDTD method," *IEEE Trans. on Microwave Theory and Tech.*, vol. 50, no. 4, pp. 1156-1164, 2002

[73] T. Kashiwa, Y. Sendo, K. Taguchi, T. Ohtani, and Y. Kanai, "Phase velocity errors of the nonstandard FDTD method and comparison with other high-accuracy FDTD methods," *IEEE Trans. on Magnetics*, vol. 39, no. 4, pp. 2125-2128, 2003

[74] J. Shibayama, M. Muraki, R. Takahashi, J. Yamauchi, and H. Nakano, "Performance evaluation of several implicit FDTD methods for optical waveguide analyses," *Journal of Lightwave Tech.*, vol. 24, no. 6, pp. 2465–2472, 2006.

[75] V. E. Nascimento, B. H. V. Borges, and F. L. Teixeira, "Split-field PML implementations for the unconditionally stable LOD–FDTD method," *IEEE Microwave and Wireless Components Letters*, vol. 16, no. 7, pp. 398–400, Jul. 2006.

[76] E. Li, I. Ahmed, and R. Vahldieck, "Numerical dispersion analysis with an improved LOD-FDTD method," *IEEE Microwave and Wireless Components Letters*, vol. 17, no. 5, pp. 319-321, 2007.

[77] T. Weiland, "A discretization method for the solution of Maxwell's equations for six-component fields", *Electronics and Communication (AEÜ)*, vol. 31, p. 116, 1977.

[78] T. Weiland, "Time Domain Electromagnetic Field Computation with Finite Difference Methods", *International Journal of Numerical Modelling*, vol. 9, pp. 295-319, 1996.

[79] B. Krietenstein, R. Schuhmann, P. Thoma, and T. Weiland, "The perfect boundary approximation technique facing the challenge of high precision field computation", in *Proceedings of the XIX International Linear Accelerator Conference (LINAC'98)*, Chicago, USA, 1998, pp. 860-862.

[80] M. Walter, I. Munteanu, "FIT for EMC," *Proc. of the 17th International Zurich Symposium on Electromagnetic Compatibility*, 2006, pp. 15-17.

[81] A. Ruehli, "Equivalent circuit models for three dimensional multiconductor systems," *IEEE Trans. on Microwave Theory and Tech.*, vol. 22, no. 3, pp. 216-221, Mar. 1974.

[82] A. Ruehli and H. Heeb, "Circuit models for three-dimensional geometries including dielectrics," *IEEE Trans. on Microwave Theory and Tech.*, vol. 40, no. 7, pp. 1507-1516, July 1992.

[83] C. Wollenberg and A. Gurisch, "Analysis of 3-D interconnect structures with PEEC using SPICE," *IEEE Trans. on Electromag. Compat.*, vol. 41, no. 4, pp. 412-417, Nov. 1999.

[84] Y. Cao, Z.-F. Li, J.-F Mao, J.-F Mao, "A PEEC with a new capacitance model for circuit simulation of interconnects and packaging structures," *IEEE Trans. on Microwave Theory and Tech.*, vol. 48, no. 9, pp. 1435-1442, Sep. 2000.

[85] D. Gope, A. Ruehli, V. Jandhyala, "Solving low-frequency EM-CKT problems using the PEEC Method," *IEEE Trans. on Adv. Packaging*, vol. 30, no. 2, pp. 313-320, may 2007.

[86] G. Antonini, "PEEC capacitance extraction of 3-D interconnects," *Science, Measurement & Technology, IET*, vol. 1, no. 4, pp. 201-209, July 2007.

[87] P. B. Johns and R. L. Beurle, "Numerical solution of 2-dimensional scattering problems using a transmission-line matrix," *Proc. Inst. Elec. Eng.*, vol. 118, no. 9, pp. 1203-1208, 1971.

[88] P.B. Johns, "A symmetrical condensed node for the TLM method," *IEEE Trans. on Microwave Theory and Tech.*, vol. 35, no. 4, pp. 370-377, 1987.

[89] N. R. S. Simons and E. Bridges, "Method for modelling free space boundaries in TLM simulations," *Electronics Letters*, vol. 26, no. 7. pp. 453-155, Mar., 1990.

[90] N. R. S. Simons and E. Bridges, "Application of the TLM method to two-dimensional scattering problems." *Int. J. Num. Modelling: Electron. Network Devices, Fields,* vol. 5. pp. 93-119, 1992.

[91] P. Saguet, "TLM method for the three dimensional analysis of microwave and mm-wave structures." Paper presented at Int. Workshop of the German IEEE M7T/AP Chapter on CAD Oriented Numerical Techniques for the Analysis of Microwave and MM-Wave Transmission-Line Discontinuities and Junctions, Stuttgart, Germany, Sept. 13, 1991.

[92] J. A. Morente, J. A. Porte, and M. Khalladi, "Absorbing boundary conditions for the TLM method," *IEEE Trans. on Microwave Theory and Tech.,* vol. 40, no. 11, pp. 2095-2099, 1992.

[93] Z. Chen, M. M. Ney, and W. J. R. Hoefer, "Absorbing and connecting boundary conditions for the TLM method," *IEEE Trans. on Microwave Theory and Tech*., vol. 41, pp. 2016–2024, Nov. 1993.

[94] N. Kukutsu and R. Konno, "Super absorption boundary condition for guided waves in the 3-D TLM simulation," *IEEE Microwave Guided Wave Lett.*, vol. 5, pp. 299–301, Sep. 1995.

[95] C. Eswarappa, and W.J.R. Hoefer, "Implementation of Berenger absorbing boundary conditions in TLM by interfacing FDTD perfectly matched layers," *Electronics Letters*, vol. 31, no. 15, pp. 1264-1266, 1995.

[96] N. Pena, and M. M. Ney, " Absorbing-boundary conditions using perfectly matched-layer (PML) technique for three-dimensional TLM simulations," *IEEE Trans. on Microwave Theory and Tech., vol.* 45, no. 10 pp. 1749-1755, 1997.

[97] Z. Shao, W. Hong, and H. Wu, "A Z-transform-based absorbing boundary conditions for 3-D TLM-SCN method," *IEEE Trans. on Microwave Theory and Tech.*, vol. 50, no. 1, pp. 222-225, 2002.

[98] L. de Menezes and W. J. R. Hoefer, "Modeling nonlinear dispersive media in 2D TLM," *Proc. 24th European Microwave Conf*., pp. 1739–1744, Sept. 1994.

[99] L. de Menezes and W. J. R. Hoefer, "Modeling frequency dependent dielectrics in TLM," *1994 IEEE APS Symp. and URSI Meet. Dig.*, Seattle, WA, pp. 1140-1143, Jun. 1994.

[100] L. de Menezes and W. J. R. Hoefer, "Modeling of general constitutive relationships using SCN TLM," *IEEE Trans. on Microwave Theory and Tech.*, vol. 44, pp. 854–861, June 1996.

[101] S. Hein, "Synthesis of TLM algorithms in the propagator integral framework," *2nd Int. Workshop Transmission-Line Matrix (TLM) Modeling Theory Applicat*., Tech. Univ., Munich, Germany, pp. 1–11, Oct. 1997.

[102] S. Hein, "TLM numerical solution of Bloch's equations for magnetized gyrotropic media," *Appl. Math. Modeling*, vol. 21, pp. 221–229, Apr. 1997.

[103] J. Paul, C. Christopoulos, and D. W. P. Thomas, “Generalized material models in TLM - part 1: Materials with frequency-dependent properties,” *IEEE Trans. Antennas Propagat.*, vol. 47, pp. 1528–1534, Oct. 1999.

[104] J. Paul, C. Christopoulos, and D. W. P. Thomas, “Generalized material models in TLM - part 2: Materials with anisotropic properties,” *IEEE Trans. Antennas Propagat.,* vol. 47, pp. 1535–1542, Oct. 1999.

[105] J. Paul, C. Christopoulos, and D. W. P. Thomas, “Generalized material models in TLM - part 3: Materials with nonlinear properties,” *IEEE Trans. Antennas Propagat.*, vol. 50, pp. 997–1004, Jul. 2002.

[106] N. K. Madsen, R.W. Ziolkowski, "A three-dimensional modified finite volume technique for Maxwell's equations", *Electromagnetics*, vol. 10, pp. 147-161, 1990.

[107] V. Shankar, A.H. Mohammadian, W.F. Hall, "A time-domain, finite-volume treatment for the Maxwell equations", *Electromagnetics*, vol. 10, pp. 127-145, 1990.

[108] P. Bonnet, X. Ferrières, B. Michielsen, and P. Klotz, *Time Domain Electromagnetics*, S. M. Rao, Ed., Academic Press, 1997, ch. 9, pp. 307–367.

[109] F. Bonnet and F. Poupaud, “Berenger absorbing boundary condition with time finite-volume scheme for triangular meshes,” *Appl. Numer. Math.*, vol. 25, no. 4, pp. 333–354, Dec. 1997.

[110] K. Sankaran, C. Fumeaux, and R. Vahldieck, “Cell-centered finite-volume-based perfectly matched layer for time-domain Maxwell system,” *IEEE Trans. on Microwave Theory and Tech.*, vol. 54, no. 3, Mar. 2006.

[111] K. Sankaran, C. Fumeaux, and R. Vahldieck, “Finite-volume Maxwellian absorber on unstructured grid,” *IEEE MTT-S Int. Microw. Symp. Dig*., pp. 169–172, Jun. 2006.

[112] K. Sankaran, C. Fumeaux, and R. Vahldieck, “Uniaxial and radial anisotropy models for finite-volume Maxwellian absorber,” *IEEE Trans. on Microwave Theory and Tech.*, vol. 54, no. 12, pp. 4297 – 4304, Dec. 2006.

[113] C. Fumeaux, K. Sankaran, and R. Vahldieck, “Spherical perfectly matched absorber for finite-volume 3-D domain truncation,” *IEEE Trans. on Microwave Theory and Tech.*, vol. 55, no.12, Dec. 2007

[114] D. Pinto and S. S. A. Obayya, “Accurate perfectly matched layer finite-volume time-domain method for photonic bandgap devices,” *IEEE Photonics Technology Letters*, vol. 20, no. 5, Mar. 2008.

[115] R. Holland, V.P. Cable, and L.C. Wilson, “Finite-volume time-domain (FVTD) techniques for EM scattering,” *IEEE Trans. on Electromag. Compat.*, vol. 33, no. 4, pp. 281-294, Nov. 1991.

[116] P. Bonnet, X. Ferrieres, F. Paladian, J. Grando, J. Alliot, and J. Fontaine, “Electromagnetic wave diffraction using a finite volume method,” *Electronics Letters,* vol. 33, no. 1, pp. 31-32, 1997.

[117] C. Fumeaux, D. Baumann, and R. Vahldieck, “Advanced FVTD simulations of dielectric resonator antennas and feed structures,” *Journal of the Applied Computational Electromagnetics Society*, vol. 19, pp. 155-164. 2004.

[118] D. Baumann, C. Fumeaux, P. Leuchtmann, and R. Vahldieck, “Finite volume time-domain (FVTD) modeling of a broad-band double-ridged horn antenna,” *Int. J. Numer. Modeling,* vol. 17, no. 3, pp. 285-298, 2004.

[119] P. Bonnet, X. Ferrieres, F. Paladian, J. Grando, J.C. Alliot and J. Fontaine, “Electromagnetic wave diffraction using a finite volume method,” *Electronics Letters,* vol. 33, no. 1, pp. 31-32, Jan. 1997.

[120] D. Lacour, X. Ferrieres, P. Bonnet, V. Gobin and J.C. Alliot, “Application of multi-domain decomposition method to solve EMC problem on an aeroplane,” *Electronics Letters,* vol. 33, no. 23, pp.1932-1933, Nov. 1997.

[121] D. Baumann, C. Fumeaux, and R. Vahldieck, “A novel wave-separation scheme for the extraction of S-parameters in non-TEM waveguides for the FVTD method,” *IEEE MTT-S Int. Microwave Symp. Dig.*, Long Beach, CA, Jun. 2005.

[122] D. Baumann, C. Fumeaux, and R. Vahldieck, “Field-Based Scattering-Matrix Extraction Scheme for the FVTD Method Exploiting a Flux-Splitting Algorithm,” *IEEE Trans. on Microwave Theory and Tech.*, vol. 53, no. 11, Nov. 2005.

[123] C. Fumeaux, D. Baumann, P. Leuchtmann, R. Vahldieck, “A generalized local time-step scheme for efficient FVTD simulations in strongly inhomogeneous meshes,” *IEEE Trans. on Microwave Theory and Tech.*, vol. 52, no. 3, pp. 1067-1076, Mar. 2004.

[124] A. Cangellaris, C. Lin, and K. Mei, “Point-matched time-domain finite element methods for electromagnetic radiation and scattering,” *IEEE Trans. Antennas Propagat.,* vol. 35, no. 10, pp. 1160–1173, Oct. 1987.

[125] C. Chan, J. Elson, and H. Sangani, “An explicit finite-difference time-domain method using Whitney elements,” *Proc. IEEE APS Int. Symp. Dig.*, vol. 3, pp. 1768–1771, Jul. 1994.

[126] M. F. Wong, O. Picon, and V. Hanna, “A finite element method based on Whitney forms to solve Maxwell equations in the time domain,” *IEEE Trans. on Magnetics*, vol. 31, no. 3, pp. 1618–1621, May 1995.

[127] K. Choi, S. J. Salon, K. A. Connor, L. F. Libelo, and S. Y. Hahn, “Time domain finite analysis of high power microwave aperture antennas,” *IEEE Trans. on Magnetics*, vol. 31, no. 5, pp. 1622–1625, May 1995.

[128] J. T. Elson, H. Sangani, and C. H. Chan, “An explicit time-domain method using three-dimensional Whitney elements,” *Microw. Opt. Technol. Lett.*, vol. 7, pp. 607–610, Sep. 1994.

[129] M. Feliziani and F. Maradei, “Point matched finite element-time domain method using vector elements,” *IEEE Trans. on Magnetics*, vol. 30, no. 5, pt. 2, pp. 3184–3187, Sep. 1994.

[130] M. Feliziani and F. Maradei, "An explicit-implicit solution scheme to analyze fast transients by finite elements," *IEEE Trans. on Magnetics*, vol. 33, no. 2, pp. 1452–1455, Mar. 1997.

[131] T. Yamada and K. Tani, "Finite element time domain method using hexahedral elements," *IEEE Trans. on Magnetics*, vol. 33, no. 2, pp. 1476-1479, Mar. 1997.

[132] T. Yioultsis, N. Kantartzis, C. Antonopoulos, and T. Tsiboukis, "A fully explicit Whitney element-time domain scheme with higher order vector finite elements for three-dimensional high frequency problems," *IEEE Trans. on Magnetics*, vol. 34, no. 5, pp. 3288-3291, Sep. 1998.

[133] W. Artuzi, "Finite element time domain method using piecewise constant basis functions," *Proc. IEEE MTT-S Int. Microw. Optoelectr. Conf.*, vol. 2, pp. 1029–1032, 2003.

[134] O. Zienkiewicz and R. Taylor, *The Finite Element Method, 4th ed.*, New York: McGraw-Hill, 1988.

[135] R. Fernades and G. Fairweather, "An alternating direction Galerkin method for a class of second order hyperbolic equations in two space dimensions," *SIAM J. Numer. Anal.*, vol. 28, no. 5, pp. 1265–1281, 1991.

[136] G. Mur, "The finite-element modeling of three-dimensional time-domain electromagnetic fields in strongly inhomogeneous media," *IEEE Trans. on Magnetics*, vol. 28, no. 3, pp. 1130–1133, Mar. 1992.

[137] J. F. Lee and Z. Sacks, "Whitney elements time domain (WETD) methods," *IEEE Trans. on Magnetics*, vol. 31, no. 5, pp. 1325–1329, May 1995.

[138] S. D. Gedney and U. Navsariwala, "An unconditionally stable finite-element time-domain solution of the vector wave equation," *IEEE Microw. Guided Wave Lett.*, vol. 5, no. 5, pp. 332–334, May 1995.

[139] J. Lee, R. Lee, and A. Cangellaris, "Time-domain finite-element methods," *IEEE Trans. Antennas Propagat.*, vol. 45, no. 3, pp. 430–442, Mar. 1997.

[140] W. P. Capers, Jr., L. Pichon, and A. Razek, "A 3-D finite element method for the modeling of bounded and unbounded electromagnetic problems in the time domain," *Int. J. Numer. Modeling*, vol. 13, pp. 527–540, 2000.

[141] J. M. Jin, *The Finite Element Method in Electromagnetics, 2nd ed.*, New York: Wiley, 2002.

[142] F. Edelvik, G. Ledfelt, P. Lotstedt, and D. Riley, "An unconditionally stable subcell model for arbitrarily oriented thin wires in the FETD method," *IEEE Trans. Antennas Propagat.*, vol. 51, no. 8, pp. 1797–1805, Aug. 2003.

[143] D. Jiao, J. M. Jin, E. Michielssen, and D. Riley, "Time-domain finite-element simulation of three-dimensional scattering and radiation problems using perfectly matched layers," *IEEE Trans. Antennas Propagat.*, vol. 51, no. 2, pp. 296–305, Feb. 2003.

[144] T. Rylander and J. M. Jin, “Perfectly matched layers in three dimensions for the time-domain finite element method,” *Proc. IEEE APS Int. Symp. Dig.*, Monterey, CA, Jun. 2004, vol. 4, pp. 3473–3476.

[145] Z. Lou and J. M. Jin, “Modeling and simulation of broadband antennas using the time-domain finite element method,” *IEEE Trans. Antennas Propagat.*, vol. 53, no. 12, pp. 4099–4110, Dec. 2005.

[146] A. Peterson and J. M. Jin, “A three-dimensional time-domain finite element formulation for periodic structures,” *IEEE Trans. Antennas Propagat.*, vol. 54, no. 1, pp. 12–19, Jan. 2006.

[147] Z. Lou and J. M. Jin, “A novel dual-field time-domain finite-element domain-decomposition method for computational electromagnetics,” *IEEE Trans. Antennas Propagat.*, vol. 54, no. 6, pp. 1850–1862, Jun. 2006.

[148] Z. Lou and J. M. Jin, “A new explicit time-domain finite-element method based on element-level decomposition,” *IEEE Trans. Antennas Propagat.*, vol. 54, no. 10, pp. 2990–2999, Oct. 2006.

[149] H. Ali and G. Costache, “Finite-element time-domain analysis of axisymmetrical radiators,” *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 272–275, Feb. 1994.

[150] S. Caorsi and G. Cevini, “Assessment of the performances of first- and second-order time-domain ABC’s for the truncation of finite element grids,” *Microwave Opt. Technol. Lett.*, vol. 38, no. 1, pp. 11–16, Jul. 2003.

[151] D. Jiao, M. Lu, E. Michielssen, and J. M. Jin, “A fast time-domain finite element-boundary integral method for electromagnetic analysis,” *IEEE Trans. Antennas Propagat.*, vol. 49, no. 10, pp. 1453–1461, Oct. 2001.

[152] M. Kuzuoglu and R. Mittra, “Investigation of nonplanar perfectly matched absorbers for finite-element mesh truncation,” *IEEE Trans. Antennas Propagat.*, vol. 45, no. 3, Mar. 1997.

[153] V. Mathis, “An anisotropic perfectly matched layer-absorbing medium in finite element time domain method for Maxwell’s equations,” *Proc. IEEE Antennas and Propagation Soc. Int. Symp.*, vol. 2, pp. 680–683, Jul. 1997.

[154] D. Jiao and J. M. Jin, “An effective algorithm for implementing perfectly matched layers in time-domain finite-element simulation of open-region EM problems,” *IEEE Trans. Antennas Propagat.*, vol. 50, pp. 1615–1623, Nov. 2002.

[155] D. Jiao, J. M. Jin, E. Michielssen, and D. J. Riley, “Time-domain finite element simulation of three-dimensional scattering and radiation problems using perfectly matched layers,” *IEEE Trans. Antennas Propagat.*, vol. 51, no. 2, pp. 296–305, Feb. 2003.

[156] H. P. Tsai, Y. Wang, and T. Itoh, “An unconditionally stable extended (USE) finite-element time-domain solution of active nonlinear microwave circuits using perfectly matched layers,” *IEEE Trans. Microwave Theory Tech.*, vol. 50, no. 10, pp. 2226–2232, Oct. 2002.

[157] T. Rylander and J.-M. Jin, "Conformal perfectly matched layers for the time domain finite element method*," IEEE AP-S Int. Symp. Digest*, Jun. 2003, vol. 1, pp. 698–701.

[158] T. Rylander and J.-M. Jin, "Perfectly matched layer for the time domain finite element method applied to Maxwell's equations," *J. Comp. Phys.*, vol. 200, pp. 238–250, May 2004.

[159] T. Rylander and J.-M. Jin, "Perfectly matched layer in three dimensions for the time-domain finite element method applied to radiation problems," *IEEE Trans. Antennas Propagat.*, vol. 53, no. 4, pp. 1489–1499, Apr. 2005.

[160] S. Wang, R. Lee, and F. L. Teixeira, "Anisotropic-medium PML for vector FETD with modified basis functions," *IEEE Trans. Antennas Propagat.*, vol. 54, no. 1, pp. 20–27, Jan. 2006.

[161] S. M. Rao, T. K. Sarkar, and S. A. Dianat, "A novel technique to the solution of transient electromagnetic scattering from thin wires," *IEEE Trans. Antennas Propagat.*, vol. 34, pp. 630–634, May 1986.

[162] A. R. Bretones, R. G. Martin, and A. Salinas, "DOTIG1, a time-domain numerical code for the study of the interaction of electromagnetic pulses with thin-wire structures," *COMPEL*, vol. 8, no. 1, pp. 39–61, 1989.

[163] S. M. Rao and D. R. Wilton, "Transient scattering by conducting surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, vol. 39, no. 1, pp. 56–61, Jan. 1991.

[164] B. P. Rynne, "Time domain scattering from arbitrary surfaces using the electric field integral equation," *J. Electromagn. Waves Appl.*, vol. 5, pp. 93–112, 1991.

[165] D. A. Vechinski and S. M. Rao, "A stable procedure to calculate the transient scattering by conducting surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, vol. 40, pp. 661–665, June 1992.

[166] P. J. Davies, "Time-Marching numerical schemes for the electric field integral equation on a straight thin wire," *Advances in Computational Mathematics*, 2, 1994.

[167] G. Manara, A. Monorchio, R. Reggiannini, "A space-time discretization criterion for a stable time-marching solution of the electric field integral equation," *IEEE Trans. Antennas Propagat.*, vol. 45, no. 3, March 1997.

[168] G. Cerri, R. de Leo, R. de Rentiis, V.M. Primiani, "ESD field penetration through slots into shielded enclosures: a time domain approach," *IEEE Trans. on Electromag. Compat.*, vol. 39, no. 4, pp. 377-386, Nov. 1997.

[169] M. J. Bluck and S. P. Walker, "Time-domain BIE analysis of large three-dimensional electromagnetic scattering problems," *IEEE Trans. Antennas Propagat.*, vol. 45, pp. 894–901, May 1997.

[170] S. M. Rao and T. K. Sarkar , "Time domain modeling of two dimensional conducting cylinders utilizing an implicit scheme," *Microwave Opt. Technol. Lett.*, vol. 15, pp. 342–347, Aug. 1997.

[171] S. M. Rao and T. K. Sarkar, “Transient analysis of electromagnetic scattering from wire structures utilizing an implicit time domain integral equation technique,” *Microwave Opt. Technol. Lett*., vol. 16, pp. 66–69, Jan. 1998.

[172] S. M. Rao and T. K. Sarkar, “An efficient method to evaluate the time-domain scattering from arbitrarily shaped conducting bodies,” *Microwave Opt. Technol. Lett*., vol. 17, pp. 321–325, Apr. 1998.

[173] M. D. Pocock, M. J. Bluck, and S. P. Walker, “Electromagnetic scattering from 3-D curved dielectric bodies using time-domain integral equations,” *IEEE Trans. Antennas Propagat.*, vol. 46, pp. 1212–1219, Aug. 1998.

[174] S. M. Rao, D. A. Vechinski, and T. K. Sarkar, “Transient scattering by conducting cylinders—Implicit solution for transverse electric case,” *Microwave Opt. Technol. Lett.*, vol. 21, pp. 129–134, Apr. 1999.

[175] S. M. Rao and T. K. Sarkar, “Implicit solution of time domain integral equations for arbitrarily shaped dielectric bodies,” *Microwave Opt. Technol. Lett.,* vol. 21, pp. 201–205, May 1999.

[176] D. S. Weile, B. Shanker, E. Michielssen, “An accurate scheme for the numerical solution of the time domain electric field integral equation,” *Proc. of IEEE Int. Symp. on Antennas and Propagation*, vol. 4, pp. 516-519, July 2001.

[177] P. Argus, H. Singer, “New basis functions for connections between wire and surface structures for time domain method of moments,” *Proc. IEEE Int. Symp. on Electromagnetic Compatibility*, vol. 1, pp. 463 – 468, Aug. 2002.

[178] S. M. Rao and T. K. Sarkar, “Numerical solution of time domain integral equations for arbitrary shaped conductor/dielectric composite bodies,” *IEEE Trans. Antennas Propagat.,* vol. 50, no. 12, pp. 1831–1837, Dec. 2002.

[179] B. Shanker, A. A. Ergin, M. Lu, and E. Michielssen, “Fast analysis of transient electromagnetic scattering phenomena using the multilevel plane wave time domain algorithm,” *IEEE Trans. Antennas Propagat.,* vol. 51, no. 3, pp. 628-641, March 2003.

[180] R.A. Wildman, G. Pisharody, D.S. Weile, S. Balasubramaniam, and E. Michielssen, “An accurate scheme for the solution of the time-domain Integral equations of electromagnetics using higher order vector bases and band limited extrapolation,” *IEEE Trans. Antennas Propagat.,* vol. 52, no. 11, pp. 2973-2984, Nov. 2004.

[181] Y.S. Chung, T.K. Sarkar, B.H. Jung, M. S. Palma, Z. Ji, S. Jang, and K. Kim, “Solution of time domain electric field Integral equation using the Laguerre polynomials,” *IEEE Trans. Antennas Propagat.,* vol. 52, no. 9, pp. 2319-2328, Sep. 2004.

[182] D.S. Weile, G. Pisharody, N.W. Chen, B. Shanker, E. Michielssen, “ A novel scheme for the solution of the time-domain integral equations of electromagnetics,” *IEEE Trans. Antennas Propagat.,* vol. 52, no. 1, pp. 283-295, Jan. 2004.

[183] Z. Ji, T.K. Sarkar, B.H. Jung, Y.S. Chung, M.S. Palma, and M. Yuan, “A stable solution of time domain electric field Integral equation for thin-wire antennas using the Laguerre polynomials,” *IEEE Trans. Antennas Propagat.,* vol. 52, no. 10, pp. 2641-2649, Oct. 2004.

[184] B. Jung, T. Sarkar, Y. Chung, M. Palma, Z. Ji, S. Jang, and K. Kim, "Transient electromagnetic scattering from dielectric objects using the electric field Integral equation with Laguerre polynomials as temporal basis functions," *IEEE Trans. Antennas Propagat.*, vol. 52, no. 9, pp. 2329-2340, Sep. 2004.

[185] S. Wang, S. Chai, J. Mao, "A method to accelerate the MOT arithmetic in TD-MOM EFIE," Microwave Conference Proceedings, 2005. APMC 2005. Asia-Pacific Conference Proceedings, vol. 4, Dec. 2005.

[186] R.A. Lemdiasov, A.A. Obi, and R. Ludwig, "Time domain formulation of the method of moments for inhomogeneous conductive bodies at low frequencies," *IEEE Trans. Antennas Propagat.*, vol. 54, no. 2, pp. 706-714, Feb. 2006.

[187] G. Guarnieri, S. Selleri, G. Pelosi, C. Dedeban and C. Pichot, "Innovative basis and weight functions for wire junctions in time domain moment method," *IEE Proceedings - Microwaves, Antennas and Propagation*, vol. 153, no. 1, pp. 61-66, Feb. 2006.

[188] F.P. Andriulli, H. Bagsi, F. Vipiana, G. Vecchi, and E. Michielssen, " A marching-on-in-time hierarchical scheme for the solution of the time domain electric field integral equation," *IEEE Trans. Antennas Propagat.*, vol. 55, no. 12, pp. 3734-3738, Dec. 2007.

[189] L.J. Li, L. Li, C. Gao, X. Gu, and Z. Zhao, "Numerical Treatment for Time Domain Integral Equations of Thin Wire Structures in Half-Space Configuration," *IEEE Trans. on Magnetics*, vol. 44, no. 6, pp. 774-777, June 2008.

[190] G. Pisharody, and D.S. Weile, "An accurate solution to time domain integral equations for inhomogeneous dielectric bodies using higher-order volume bases," *Proc. of IEEE Int. Symp.on Antennas and Propagation*, pp. 2805 – 2808, July 2006.

[191] Christian Hafner, *The Generalized Multipole Technique for Computational Electromagnetics*, Artech House, 1990.

[192] W. V. T. Rusch and P. D. Potter, *Analysis of Reflector Antennas*, New York: Academic, 1970, pp. 92-93.

[193] C. A. Balanis, *Antenna Theory Analysis and Design, Third edition*, John Wiley & Sons, Inc., 2005.

[194] A. D. Yaghjian, "Equivalence of surface current and aperture field integrations for reflector antennas," *IEEE Trans. Antennas Propagat.*, vol. 32, no. 12, Dec. 1984.

[195] J. F. Kauffman, W. F. Croswell, and L. J. Jowers, "Analysis of the radiation patterns of reflector antennas," *IEEE Trans. Antennas Propagat.*, vol. 24, no. 1, pp. 53-65, Jan. 1976.

[196] Y. Rahmat-Samii and V. Galindo, "Shaped reflector antenna analysis using the Jacobi-Bessel series," *IEEE Trans. Antennas Propagat.*, vol. 28, pp. 425-435, July 1980.

[197] T. F. Eibert, "Modeling and design of offset parabolic reflector antennas using physical optics and multilevel fast multipole method accelerated method of moments," *Multi-optic Conference, 2006. INMIC '06, IEEE*, pp. 377-382, Dec. 2006.

[198] A. Boag, "A fast physical optics (FPO) algorithm for high frequency scattering," *IEEE Trans. Antenna Propagat.*, vol. 52, no. 1, Jan. 2004.

[199] E. Y. Sun and W. V. T. Rusch, “Time-domain physical-optics,” *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 9-15, Jan. 1994.

[200] R. Mittra, Y. Rahmat-Samii, V. Galindo-Israel, R. Norman, "An efficient technique for the computation of vector secondary patterns of offset reflectors," *IEEE Trans. Antennas Propagat.*, vol. 27, pp. 294-304, May 1979.

[201] Y. Rahmat-Samii. "A comparison between go/aperture-field and physical optics methods for offset reflectors," *IEEE Trans. Antennas and Propagat*., vol. 32, pp. 301-306, Mar. 1984.

[202] G. A. Deschamps, “Ray techniques in electromagnetics,” *Proc. IEEE*, vol. 60, pp. 1022-1035, Sept. 1972.

[203] J. B. Keller, “Geometrical theory of diffraction,” *Journal of Optical Society of America*, 52(2), 1962.

[204] D. L. Hutchins, “Asymptotic series describing the diffraction of a plane wave by a two-dimensional wedge of arbitrary angle,” Ph.D. Dissertation, The Ohio State University, Dept. of Electrical Engineering, 1967.

[205] C. A. Balanis and L. Peters, Jr., “Analysis of aperture radiation from an axially slotted circular conducting cylinder using GTD,” *IEEE Trans. Antennas Propagat*., vol. 17, no. 1, pp. 93-97, Jan. 1969.

[206] G. L. James and V. Kerdemelidis, “Reflector antenna radiation pattern analysis by equivalent edge currents,” *IEEE Trans. Antenna Propagat*., vol. 21, no. 1, pp. 19-24, Jan. 1973.

[207] C. A. Mentzer and L. Peter, Jr., “A GTD analysis of far-out sidelobes of Cassegrain antennas,” *IEEE Trans. Antenna Propagat*., vol. 23, no. 5, pp. 702-709, Sep. 1975.

[208] P .H. Pathak and R. G. Kouyoumjian, “An analysis of the radiation from aperture on curved surfaces by the geometrical theory of diffraction,” *Proc. IEEE*, vol. 62, no. 11, pp. 1438-1447, Nov. 1974.

[209] A. W. Rudge, “Offset-parabolic-reflector antennas: A review,” *Proc. IEEE*, vol. 66, no. 12, pp. 1592-1618, Dec. 1978.

[210] D. P. Bouche, F. A. Molinet and R. Mittra, “Asymptotic and hybrid techniques for electromagnetic scattering,” *Proc. IEEE*, vol. 81, no. 12, pp. 1658-1684, Dec. 1993.

[211] R. C. Kouyoumjian and P. H. Pathak, “A geometrical theory of diffraction for an edge in a perfectly conducting surface,” *Proc. IEEE*, vol. 62, pp. 1448-1461, Nov. 1974.

[212] S. W. Lee and G. A. Deschamps, “A uniform asymptotic theory of electromagnetic diffraction by a curved wedge,” *IEEE Trans. Antennas Propagat*., vol. 24, pp. 25-34, Jan. 1976.

[213] R. W. Ziolkowski and G. A. Deschamps, “Asymptotic evaluation of high frequency fields near a caustic: An introduction to Maslov’s method,” *Radio Sci*., vol. 19, no. 4, pp. 1001-1025, Jul.-Aug. 1984.

[214] Y. Kravtsov and Y. Orlov, "Caustics, catastrophes and wave fields," *Sov. Phys.-Usp.*, vol. 26, pp. 1039-1058, Dec. 1983.

[215] H. Ikuno and L. B. Felsen, "Complex ray interpretation of reflection from concave-convex surfaces," IEEE *Trans. Antennas Propagat*., vol. 36, pp. 1206-1211, 1988.

[216] P. Y. Ufimtsev, "Elementary edge waves and the physical theory of diffraction," *Electromagnetics*, vol. 11, pp. 125-160, 1991.

[217] R. Mittra, Y Rahmat-Samii, and W. L. Ko, "Spectral Theory of Diffraction," *Appl. Phys*., vol. 10, pp. 1-13, 1976.

[218] Y. Rahmat-Samii and R. Mittra, "A Spectral Domain interpretation of High-Frequency Diffraction Phenomena," *IEEE Trans. Antennas Propagat.*, vol. 25, pp. 676-687, Sept. 1977.

[219] R. G. Kouyoumjian and P. H. Pathak, "A Uniform Geometrical Theory of Diffraction for an Edge in A Perfectly Conducting Surface," *Proc. IEEE*, vol. 62, pp. 1448–1461, Nov. 1974.

[220] T.-K. Lee, S.-Y. Lee and J.-W. Ra, "A hybrid finite element-boundary element method and its application to inverse scattering," *Proc. IEEE Antennas and Prop. Symp.*, vol. 3, pp. 1628-1631, June 1989.

[221] W. E. Boyse and A. A. Seidl, "A hybrid finite element and moment method for electromagnetic scattering from inhomogeneous objects," *Proc. 7th Ann. Rev. Progress Appl. Computat. Electromag.*, Mar. 1991.

[222] J. M. Jin and John L. Volakis, "Electromagnetic scattering by and transmission through a three dimensional slot in a thick conducting plane," *IEEE Trans. Antennas Propagat*., vol. 39, pp. 543-550, April 1991.

[223] K. D. Paulsen, D. R. Lynch, and J. W. Strohbehn, "Three-dimensional finite, boundary, and hybrid element solutions of the Maxwell's equations for lossy dielectric media," *IEEE Trans. on Microwave Theory and Tech.*, vol. 36, pp. 682-693, April 1988.

[224] J. Shen and A. Kost, "Hybrid FE-BE method for EMC problems in power cable systems," *IEEE Trans. on Magnetics*, vol. 32, no. 3, pp. 1493-1496, May 1996.

[225] Y. Ji, J. Chen, T.H. Hubing, J.L. Drewniak, "Application of a hybrid FEM/MOM method to a canonical PCB problem," Proc. *1999 IEEE Int. Symp. Electromagn. Compat*. Seattle, WA, August 1999, pp. 91-96.

[226] Y. Ji, T. H. Hubing, "EMAP5: A 3D hybrid FEM/MOM code," *Journal of the Applied Computational Electromagnetics Society*, vol. 15, no. 1, March 2000, pp. 1-12.

[227] K. Zhao, M. Vouvakis and J.-F. Lee, "Solving electromagnetic problems using a novel symmetric FEM-BEM approach," *IEEE Trans. on Magnetics*, vol. 42, no. 4, pp. 583-586, April 2006.

[228] Y. Ji, H. Wang and T. Hubing, "A novel preconditioning technique and comparison of three formulations for hybrid FEM/MOM methods," *Journal of the Applied Computational Electromagnetics Society*, vol. 15, no. 2, Aug. 2000, pp. 103-114.

[229] W. D. Burnside, C. L. Yu, and R. J. Marhefka, "A technique to combine the geometrical theory of diffraction and the moment method", *IEEE Trans. Antennas Propagat.*, vol. 23, no. 4, pp. 551-558, July 1975.

[230] G. A. Thiele and T. H. Newhouse, “A hybrid technique for combining moment methods with the geometrical theory of diffraction,” *IEEE Trans. Antennas Propagat.*, vol. 23, no. 1, pp. 62–69, Jan. 1975.

[231] U. Jakobus and F. M. Landstorfer, “A combination of current-and ray-based techniques for the efficient analysis of electrically large scattering problems,” *Proc. 13th Annu. Rev. Progress Appl. Computat. Electromagn.*, Monterey, CA, Mar. 1997, pp. 748–755.

[232] I. P. Theron, D. B. Davidson, and U. Jakobus, “Extensions to the hybrid method of moments/uniform GTD Formulation for sources located close to a smooth convex surface,” *IEEE Trans. Antennas Propagat*., vol. 48, no. 6, pp. 940 – 945, June 2000.

[233] I. P. Theron, U. Jakobus, I. B. Davidson, and F. J. Meyer, “Recent progress on moment method / UTD hybridization,” *Proc. of PIERS98 - Progress in Electromagnetics Research Symposium*, Nantes, France, vol. 1, p. 459, July 1998.

[234] H. J. Bilow, “Scattering by an infinite wedge with tensor impedance boundary conditions - a moment method/physical optics solution for currents,” *IEEE Trans. Antennas Propagat*., vol. 39, pp. 761-773, June 1991.

[235] L. N. Medgyesi-Mitschang and D.-S. Wang, “Hybrid solutions for scattering from perfectly conducting bodies of revolution,” *IEEE Trans. Antennas Propagat*., vol. 31, p. 570, 1983.

[236] M. Kaye, P. K. Murthy, and G. A. Thiele, “An iterative method for solving scattering problems,” *IEEE Trans. Antennas Propagat*., vol. 33, pp. 1272-1279, 1985.

[237] D. S. Wang, “Current-based hybrid analysis for surface wave effects on large scatterers,” *IEEE Trans. Antennas Propagat*., vol. 39, pp. 839-850, June 1991.

[238] C. S. Kim and Y. Rahmat-Samii, “Low profile antenna study using the physical optics hybrid method (POHM),” *Proc. IEEE Int. Symp. Antennas Propagat. Soc. Meet*., London, ON, Canada, June 1991.

[239] U. Jakobus and F. M. Landstorfer, “Improvement of the PO-MM hybrid method by accounting for effects of perfectly conducting wedges,” *IEEE Trans. Antennas Propagat*., vol. 43, pp. 1123-1129, Oct. 1995.

[240] R. E. Hodges and Y. Rahmat, “An iterative current-based hybrid method for complex structures,” *IEEE Trans. Antennas Propagat*., vol. 45, pp. 265-276, Feb. 1997.

[241] Johan Edlund, “A parallel, iterative method of moments and physical optics hybrid solver for arbitrary surfaces,” UPPSALA University, Department of Information Technology, August, 2001.

[242] D. B. Davidson, I. P. Theron, U. Jakobus, F. M. Landstorfer, F. J. C. Meyer, J. Mostert, J. J. Van Tonder, “Recent progress on the antenna simulation program FEKO”, *Proc. of the 1998 South African Symposium on Communications and Signal Processing, 1998. COMSIG '98*, pp. 427-430, 7-8 Sept. 1998.

[243] J. M. Jin, S. Ni, and S. W. Lee, “Hybridization of SBR and FEM for scattering by large bodies with cracks and cavities,” *IEEE Trans. Antennas Propagat*., vol. 43, pp. 1130-1139, Oct. 1995.

[244] Dong-Ho Han, A. C. Polycarpou, C. A. Balanis, “FEM-based hybrid methods for the analysis of antennas on electrically large structures,” *Radio and Wireless Conference, 2000. RAWCON 2000*, pp. 59-61.