TECHNICAL REPORT: CVEL-13-045

Maximum Radiated Emission Calculator:
I/O Coupling Algorithm

Chentian Zhu and Dr. Todd Hubing
Clemson University

August 24, 2013
**Table of Contents**

Abstract ..................................................................................................................................................... 3

1. Introduction ....................................................................................................................................... 3

2. Description of Algorithm .................................................................................................................. 4
   2.1 The Coupling Algorithm ............................................................................................................ 4
       2.1.1 I/O coupling model ............................................................................................................. 4
       2.1.2 Inductive Coupling ........................................................................................................... 5
       2.1.3 Capacitive Coupling .......................................................................................................... 5
       2.1.4 Total Coupling .................................................................................................................... 6
       2.1.5 Mutual Inductance and Capacitance .................................................................................. 6
   2.2 Thevenin Equivalent Algorithm ................................................................................................. 8
   2.3 Maximum Radiated Emission Estimation Algorithm ................................................................ 9
       2.3.1 Board-source-cable geometry ............................................................................................. 9
       2.3.2 Maximum Radiated Emission Estimation ........................................................................ 10
   2.4 Assumptions Made in this Derivation ...................................................................................... 11
   2.5 Limitations due to Implementation .......................................................................................... 11

3. Conclusion ...................................................................................................................................... 11

4. Appendix (Java codes) .................................................................................................................... 13
   4.1 Subroutine calcCeCo(epsr) ........................................................................................................ 13
   4.2 Subroutine calcMutual() .......................................................................................................... 13
   4.3 Subroutine findFmax(f) ............................................................................................................ 13
   4.4 Subroutine calcEM() ................................................................................................................ 14

References ............................................................................................................................................... 12
Abstract

The Maximum Radiated Electromagnetic Emissions Calculator (MREMC) is a software tool that allows the user to calculate the maximum possible radiated emissions that could occur due to specific source geometries on a printed circuit board. This report describes the I/O coupling algorithm, which determines the maximum possible radiated emissions that could occur due to coupling from a source signal on one trace to another (I/O) trace that could carry the coupled signal off the board. The methods used, calculations made, and implementation details are described.

1. Introduction

High frequency signals on one circuit board trace can couple to input/output (I/O) traces that carry the coupled energy away from the board. The common-mode currents induced on cables attached to I/O nets can result in significant radiated emissions. The I/O coupling EMI calculator was developed to calculate the maximum possible radiated emissions from structures like this. The calculator utilizes formulas for crosstalk between PCB traces described by Gupta [4] and expressions for the maximum radiated emissions from PCB-cable structures developed by Deng [2]. This report is an extension of the method described by Su [1] and is intended to provide details of the implementation sufficient to allow others to develop their own version of this calculator.

Fig. 1. I/O Coupling model: (a), top view, (b), section view.

Two parallel sections of microstrip circuit board traces are illustrated in Fig. 1. The cross-sectional view in Fig. 1(b) shows that both traces have a width, $a$, a height, $h$, and edge-to-edge separation, $s$. The board length, $L$, board width, $W$, relative dielectric constant, $\varepsilon_r$, coupling length, $l_{coupling}$, and I/O trace length, $l_{trace}$, are the other geometrical parameters required for this calculation. $V_{Signal}$ and $R_L$ represent the signal source voltage and the load resistance of the signal trace respectively. $R_{NE}$ is the near end resistance of the I/O trace. The I/O cable length is unspecified, but board is assumed to be 1 meter over a ground plane, as it would be in most radiated emissions tests [2].

The calculator calculates the maximum radiated electric field at a distance of 3 meters from the board and plots the results in $dB \mu V/m$ from 0 to 100 MHz as shown in Fig. 2.
2. Description of Algorithm

The algorithm used by the calculator can be broken into two main parts. The first is to determine the equivalent common-mode (CM) source based on the trace geometry. The second is to determine the maximum radiated emissions based on the CM source and cable-board geometry. To determine the CM source, the total voltage coupled to the victim circuit is determined by the coupling algorithm and the Thevenin equivalent algorithm. After the CM source is obtained, the maximum radiated emissions are then estimated by the estimation algorithm.

2.1 The Coupling Algorithm

2.1.1 I/O coupling model

Fig. 3(a) shows the coupling model, which can be represented more simply as shown in Fig. 3(b). $V_S$, $Z_L$ and $Z_{NE}$ are the same as $V_S$, $R_L$ and $R_{NE}$ indicated in Fig. 1. Note that in this calculator, $Z_L$ and $Z_{NE}$ only support resistive input. $Z_{FE}$ is the far-end load of the I/O trace, representing the input impedance of the antenna formed by the I/O cable being driven against the wide PCB ground plane. $L_m$ represents the mutual inductance between the two trace-ground loops. $C_m$ represents the mutual capacitance between the two traces. Inductive coupling occurs when changing current in the signal trace induces a voltage on the I/O trace through $L_m$. Similarly, the capacitive coupling occurs when a changing voltage on the signal trace induces a current on the I/O trace through $C_m$. 

![MREMC plot example](image-url)
2.1.2 Inductive Coupling

Fig. 3(c) is the lumped-element circuit model for the inductive coupling. The I/O trace and return plane are represented as a transmission line of length \( l \). \( V_{\text{ind}} \) represents the induced electromotive force due to inductive coupling, which is given by

\[
V_{\text{ind}} = -j\omega L_m I_{\text{source}}
\]

where \( I_{\text{source}} \) is the current on the signal which can be obtained by \( V_{\text{Signal}} \) / \( Z_L \). Note that the self inductance of the signal trace loop is ignored since at a frequency where the loop inductance matters, the trace usually has a matched load. In the subroutine \( \text{calcEM}() \), only the magnitude of the \( V_{\text{ind}} \) is calculated,

\[
|V_{\text{ind}}| = 2\pi f L_m I_{\text{source}}.
\]

2.1.3 Capacitive Coupling

Fig. 3(d) is the lumped-element circuit model for the capacitive coupling. An independent current source, \( I_{\text{cap}} \), represents the induced current due to capacitive coupling, which is given by

\[
I_{\text{cap}} = j\omega C_m V_{\text{signal}} \approx j\omega C_m Z_L I_{\text{source}}.
\]

In the subroutine \( \text{calcEM}() \), only the magnitude of the \( I_{\text{cap}} \) is calculated. Then magnitude of \( V_{\text{cap}} \) is obtained by

\[
|V_{\text{cap}}| = |I_{\text{cap}}| Z_{\text{NE}} = 2\pi f C_m Z_L Z_{\text{NE}} I_{\text{source}}.
\]
2.1.4 Total Coupling

Assuming the lines are weakly coupled, the maximum possible coupling is a linear combination of contributions due to the inductive and capacitive coupling [3]. The maximum voltage induced in the victim circuit is the sum of the two coupled voltages,

\[ |V_{total}| = |V_{ind} + V_{cap}| = \left| j\omega \left[ -L_m + C_m Z_{LNE} \right] I_{source} \right| = |V_{ind}| + |V_{cap}|. \]  

(5)

2.1.5 Mutual Inductance and Capacitance

The mutual inductance \( L_m \) and mutual capacitance \( C_m \) are required to calculate the induced voltages. The subroutine \( \text{calcMutual()} \) calculates \( L_m \) and \( C_m \) by [4]

\[ C_m = \frac{1}{2} \left[ C_o(\varepsilon_r) - C_e(\varepsilon_r) \right] l_{coupling} \]  

(6)

\[ L_m = \frac{\mu_0 \varepsilon_0}{2} \left[ \frac{1}{C_e(\varepsilon_r = 1)} - \frac{1}{C_o(\varepsilon_r = 1)} \right] l_{coupling} \]  

(7)

where \( C_o \) and \( C_e \) are even and odd mode capacitances per unit length respectively. \( l_{coupling} \) is the coupling length for user input. (6) and (7) only apply to symmetrical traces (traces with same width) [4]. For coupled microstrip lines, the components of the line capacitance are illustrated in Fig. 4. The algorithms to calculate are \( C_o \) and \( C_e \) are included in the subroutine \( \text{calcCeCo(epsr)} \).

![Fig. 4 Configuration of coupled microstrip line (a), general equivalent circuit (b), and breakup of even mode (c) and odd mode (d) capacitance](image)

For the even mode, the capacitance \( C_e \) is given as [4],

\[ C_e(\varepsilon_r) = C_p + C_f + C_i \]  

(8)

where,

\[ C_p = \frac{\varepsilon_0 \varepsilon_r W}{h} \]  

(9)
\[ C_f = \frac{1}{2} \left[ \frac{\sqrt{\varepsilon_r}}{cZ_0} - C_p \right] \]  
(10)

\[ C_f = \frac{C_f \sqrt{\varepsilon_r}}{1 + A(h/s) \tanh(10s/h)} \]  
(11)

where,

\[ A = \exp[-0.1 \exp(2.33 - 1.5w/h)] \]  
(12)

\[ Z_0 = \begin{cases} 
\frac{120\pi}{2\pi \sqrt{\varepsilon_r}} \ln\left(\frac{w + 0.1w}{w + 0.25w/h}\right), & w/h \leq 1 \\
\frac{120\pi}{\sqrt{\varepsilon_r}} \left[\frac{w + 1.393 + 0.667\ln\left(\frac{w + 1.444}{w + 1/3}\right)}{w + 1/3}\right]^{-1}, & w/h > 1 
\end{cases} \]  
(13)

where \( \varepsilon_{re} \) is effective relative permittivity, which can be found by [5],

\[ \varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left( \frac{1}{\sqrt{1 + 12h/a}} \right). \]  
(14)

The values are found to be accurate to within 3 percent, compared with the values obtained from [6], over the following range of parameters [4],

\[ 0.1 \leq w/h \leq 10 \quad 0.1 \leq s/h \leq 5 \quad 1 \leq \varepsilon_r \leq 18 \]

For the odd mode, the capacitance \( C_o \) is found by [4],

\[ C_o(\varepsilon_r) = C_p + C_f + C_f' + C_{ga} + C_{gd} = 0.5C_{ox} + C_{cys} \]  
(15)

where,

\[ C_{cys} = \varepsilon_0 \frac{K(k')}{K(k)} \]  
(16)

\[ C_{ox} = 4\varepsilon_0\varepsilon_r \frac{K(k_0)}{K(k_0)} \]  
(17)

where,

\[ k = \frac{s}{s + 2w} \]  
(18)

\[ k' = \sqrt{1 - k^2} \]  
(19)

\[ k_0 = \tanh\left(\frac{\pi w}{4h}\right) \coth\left[\frac{\pi}{4}\left(\frac{w + s}{h}\right)\right] \]  
(20)

\[ k_0' = \sqrt{1 - k_0^2}. \]  
(21)
The function $K(k)$ and $K(k')$ are the complete elliptic function and its complement and their ratio is given by

$$
\frac{K(k')}{K(k)} = \begin{cases} 
\frac{1}{\pi} \ln \left( \frac{2 + \sqrt{k'}}{1 - \sqrt{k'}} \right) & 0 \leq k' \leq \frac{1}{\sqrt{2}} \\
\pi \ln \left( \frac{2 + \sqrt{k'}}{1 - \sqrt{k'}} \right) & \frac{1}{\sqrt{2}} \leq k' \leq 1
\end{cases}.
$$

(22)

Same applies to $K(k_0) / K(k_0')$.

The capacitances obtained by using the above equations are accurate within 3 percent, compared with values obtained from [6], over the range of parameters [4],

$$0.1 \leq w / h \leq 10 \quad 0.1 \leq s / h \leq 4 \quad 2 \leq \varepsilon_r \leq 18$$

### 2.2 Thevenin Equivalent Algorithm

A Thevenin equivalent source model was derived to account for all of the coupling without requiring the input impedance of the attached cable to be known. The I/O trace may or may not be electrically short and is modeled as a transmission line as indicated in Fig. 5(a). The open-circuit voltage at the far end (i.e., the connector) $V_{eq}$, and the equivalent impedance looking back toward the near end from the connector $Z_{eq}$, can be readily calculated from transmission line theory yielding the Thevenin equivalent circuit in Fig. 5(b).

![Fig. 5 Thevenin equivalent model](image)

$V_{eq}$ and $Z_{eq}$ can be found by [1]

$$
V_{eq} = 2 \left( \frac{Z_0}{Z_0 + jZ_{NE} \tan \beta l} \right) \left( \frac{V_{total}}{e^{j\beta l} + e^{-j\beta l}} \right)
$$

(23)

$$
Z_{eq} = Z_0 \left( \frac{Z_{NE} + jZ_0 \tan \beta l}{Z_0 + jZ_{NE} \tan \beta l} \right)
$$

(24)

where $Z_0$ is the characteristic impedance of the transmission line, which is given in (13) and $\beta l$ is the wavenumber, which is given by

$$
\beta l = \frac{2\pi f l_{trace} \sqrt{\varepsilon_r}}{c_0}.
$$

(25)

In the subroutine `calcEM()`, the magnitude of $V_{eq}$, the real part of $Z_{eq}$ and the imaginary part of $Z_{eq}$ are calculated separately by
\[ |V_{eq}| = \left( \frac{Z_0}{\sqrt{Z_0^2 + (Z_{\text{NE}} \tan \beta l)^2}} \right) |V_{\text{total}}| \cos \beta l \]  
(26)

\[ Z_{eq,\text{real}} = \frac{Z_0^2 Z_{\text{NE}} + Z_0^2 Z_{\text{NE}} \tan \beta l^2}{Z_0^2 + (Z_{\text{NE}} \tan \beta l)^2} \]  
(27)

\[ Z_{eq,\text{imag}} = \frac{Z_0^3 \tan \beta l - Z_0^2 Z_{\text{NE}}^2 \tan \beta l}{Z_0^2 + (Z_{\text{NE}} \tan \beta l)^2} \]  
(28)

Fig. 3(a) can then be replaced by the model in Fig. 6 with the Thevenin equivalent source voltage and impedance. The new model is ready for use in the radiated emission estimation.

2.3 Maximum Radiated Emission Estimation Algorithm

2.3.1 Board-source-cable geometry

A simplified geometry representing a typical EMC test environment is shown in Fig. 7, where the PCB board is 1m above the ground. Study in [2] suggests that the peak emissions from such geometry are relatively independent of the connection point to the board and relatively insensitive to the total cable length or orientation. The parameters that matter are the vertical distance traversed by the cable and the maximum current. Also the maximum radiated electric field for this geometry can be estimated by comparing the emissions from this structure to the emissions from a thin-wire monopole above an infinite ground plane. In [2], a closed-form formula was developed to estimate the maximum radiated emissions from the antenna model in Fig. 7. This formula was enhanced in [7] to be more accurate over the larger frequency ranges.

![Fig. 7 Board-source-cable geometry](image-url)
2.3.2 Maximum Radiated Emission Estimation

In subroutine `calcEM()`, the maximum electric field at 3m as shown in Fig. 6 is calculated by [2],

\[
|E|_{\text{max}} = 20 \times I_{\text{peak}} \times f(\theta, k, l_{\text{cable}})
\]  

where \( f(\theta, k, l_{\text{cable}}) \) can be obtained by [7],

\[
f(\theta, k, l_{\text{cable}}) = \begin{cases} 
\frac{2}{\sin(\sqrt{2})} & \text{if } f \leq \frac{c_0}{2l_{\text{cable}}} \\
\frac{2}{\sin\left(\frac{c_0}{\sqrt{l_{\text{cable}}}l_{\text{cable}}}\right)} & \text{if } f > \frac{c_0}{2l_{\text{cable}}}
\end{cases}
\]

where \( l_{\text{cable}} \) is the length of the attached cable, which is set to 1m in the calculator. \( f \) is the frequency, and \( c_0 \) is the propagation velocity in free space. The equation is calculated in subroutine `finFmax()`. \( I_{\text{peak}} \) is the highest current that actually exists on the cable and is given by

\[
I_{\text{peak}} = \frac{V_{\text{eq}}}{Z_{\text{eq}} + \frac{R_{\text{min}}}{\text{board}_-\text{factor} \times \text{cable}_-\text{factor}}}
\]

where \( R_{\text{min}} \) is the input resistance (about 37 \( \Omega \)) of a resonant quarter-wave monopole. Two factors account for the effect that the finite cable length and the small board size have on this minimum resistance, which are given by,

\[
\text{board}_-\text{factor} = \begin{cases} 
\sin(2\pi l_{\text{board}}/\lambda) & \text{when } l_{\text{board}} \leq \frac{\lambda}{4} \\
1.0 & \text{otherwise}
\end{cases}
\]

\[
\text{cable}_-\text{factor} = \begin{cases} 
\sin(2\pi l_{\text{cable}}/\lambda) & \text{when } l_{\text{cable}} \leq \frac{\lambda}{4} \\
1.0 & \text{otherwise}
\end{cases}
\]

where \( l_{\text{board}} \) is the effective length of a rectangular board. It can be approximated as,

\[
l_{\text{board}} = \frac{1 + 2L/W}{2L/W} \times \sqrt{L^2 + W^2}
\]

where \( L \) and \( W \) denote the board length and width, respectively as shown in Fig. 1. Equation 31 is then calculated in subroutine `calcEM()` as,

\[
I_{\text{peak}} = \frac{V_{\text{eq}}}{\sqrt{\left(Z_{\text{eq.real}} + \frac{37}{\text{board}_-\text{factor} \times \text{cable}_-\text{factor}}\right)^2 + Z_{\text{eq.imag}}^2}}.
\]
2.4 Assumptions Made in this Derivation

1. Signal trace and I/O trace are weakly coupled.
   The induced currents and voltages in the victim circuit will induce currents and voltages back into the generator circuit. By assuming weak coupling, the currents and voltages coupled back into the generator circuit are ignored. [3]

2. Portion of the signal trace coupling to the I/O line is electrically short with a self-capacitance and self-inductance that are negligible compared to the source and load impedances. This is frequently the case, but similar equations that do not depend on the value of $Z_{FE}$ could be readily derived for longer signal traces. [1]

3. The I/O trace can be model as a lossless transmission line, which is a reasonable approximation.

4. The attached cable has negligible diameter, which is a good approximation when the cable diameter is considerably smaller than the wavelength.

2.5 Limitations Due to Implementation

1. These calculations are designed for symmetric microstrip lines.

2. The coupling algorithm provides reasonably accurate values in the following range, $0.1 \leq w / h \leq 10 \quad 0.1 \leq s / h \leq 4 \quad 2 \leq \varepsilon_r \leq 18$

3. The Estimation algorithm currently calculates emissions for a typical EMC test environment with the EUT set 1 meter above the ground and the measuring antenna located 3 meters away.

3. Conclusion

This calculator determines the maximum possible radiated emissions due to coupling from a signal trace to an I/O trace on a circuit board. It is limited to symmetric microstrip lines and assumes that the length of the coupled section is small relative to a wavelength at the highest frequency of the analysis. Applied to longer coupled sections, the calculator will overestimate the possible radiated emissions.
References


Appendix (Java codes)

The java codes contain all the subroutines are list below.

1. Subroutine calcCeCo(epsr)

```java
function calcCeCo(epsr)
{
    var carray=new Array();
    var epsre=(epsr+1)/2+(epsr-1)/2*Math.pow(1+12*t/a,-0.5);  //Eq. 14
    var z0;
    if (a<=t){
        z0=60/Math.sqrt(epsre)*Math.log(8*t/a+0.25*a/t);
    } else{
        z0=120*pi/(epsre)/(a/t+1.393+0.667*Math.log(a/t+1.444));  //Eq. 13
    }
    var A=Math.exp(-0.1*Math.exp(2.33-1.5*a/t));  //Eq. 12
    var cp=eps0*epsr*a/t;  //Eq. 9
    var cf=1/2*(Math.sqrt(epsr)/c0/z0-cp);  //Eq. 10
    var x=10^s/t;
    var tanhx=(Math.exp(x)-Math.exp(-1*x))/(Math.exp(x)+Math.exp(-1*x));
    var cfp=cf*Math.pow(epsr/epsre,0.25)/(1+A*t/s*tanhx);  //Eq. 11
    carray[0]=cp+cf+cfp;  //Eq. 8
    var k=s/(s+2*a); //Eq. 18
    var kp=Math.sqrt(1-k*k); //Eq. 19
    if (k<(1/Math.sqrt(2))
        var Ccps=eps0/pi*Math.log(2*(1+Math.sqrt(kp))/(1-Math.sqrt(kp)));  //Eq. 16
    else
        var Ccps=eps0*pi/Math.log(2*(1+Math.sqrt(kp))/(1-Math.sqrt(kp)));  //Eq. 15
    return carray;
}
```

1.1 Subroutine calcMutual()

```java
function calcMutual()
{
    var carray=calcCeCo(epsr);
    cm=0.5*(carray[1]-carray[0])*lcoupling;  //Eq. 6
    cs=0.5*(carray[1]+carray[0])*lcoupling;
    carray=calcCeCo(1);
    lm=mu0*eps0/2*(1/carray[0]-1/carray[1])*lcoupling;  //Eq. 7
    ls=mu0*eps0/2*(1/carray[0]+1/carray[1])*lcoupling;  //Eq. 24
}
```

1.2 Subroutine findFmax(f)

```java
function findFmax(f)
{
    var temp=c0/2/lcable;
    if (f<=temp)
        fmax=2/Math.sin(Math.sqrt(2));
    else
        fmax=2/Math.sin(Math.sqrt(2));
}
```
ffmax=2/Math.sin(Math.sqrt(c0/(f*1e6)/lcable)); //Eq. 30

1.3 Subroutine calcEM()

function calcEM(){
    calcMutual();
    zl=document.getElementsByName("tparr[0]").item(0).value;
    zne=document.getElementsByName("tparr[1]").item(0).value;
    var vind,vicap;
    var vtotal=new Array();
    for (i=0;i<vdm.length;i++){
        vind=2*pi*x[i]*1e6*lm*vdm[i]/zl;  //Eq.2
        vcap=2*pi*x[i]*1e6*cm*vdm[i]*zne;  //Eq.4
        vtotal[i]=Math.abs(vind-vicap);  //Eq. 5
    }
    var z0;
    if (a<=t)
        z0=60*Math.log(8*t/a+a/4/t);
    else
        z0=120*pi/(a/t+1.393+0.667*Math.log(a/t+1.444)); //Eq. 13
    var lboard=(1+2*l/w)/(2*l/w)*Math.sqrt(epsr);  //Eq. 34
    var betal,boardfactor,cablefactor,lambda;
    var veq,zant,ztot,icm,ffmax;
    var temp1,temp2,temp3;
    for (i=0;i<vtotal.length;i++){
        lambda=c0/(x[i]*1e6);
        betal=2*pi*x[i]*1e6/c0*ltrace*Math.sqrt(epsr);  //Eq. 25
        if (lambda>=lboard*4)
            boardfactor=Math.sin(2*pi*lboard/lambda);
        else
            boardfactor=1;  //Eq. 32
        if (lambda>=lcable*4)
            cablefactor=Math.sin(2*pi*lcable/lambda);
        else
            cablefactor=1;  //Eq. 33
        zant=rmin/boardfactor/cablefactor;
        veq=vtotal[i]/Math.abs(Math.cos(betal))*z0/Math.sqrt(z0*z0+Math.pow(Math.tan(betal)*zne,2)); //Eq. 26
        temp1=z0*z0*zne+zne*z0*z0*Math.pow(Math.tan(betal),2);
        temp2=Math.pow(z0,3)*Math.tan(betal)-Math.pow(zne,2)*z0*Math.tan(betal);
        temp3=z0*z0+Math.pow(zne*Math.tan(betal),2);
        zeqr=temp1/temp3; // Eq. 27
        zeqi=temp2/temp3; // Eq. 28
        ztot=Math.sqrt((zant+zeqr)*(zant+zeqr)+zeqi*zeqi);
        icm=veq/ztot;  //Eq. 35
        ffmax=findFmax(x[i]);
        y[i]=sigNumber(20*Math.log(60*icm*ffmax/r*1e6)*Math.LOG10E); //Eq. 29
    }
}