

**TECHNICAL REPORT: CVEL-13-051**

**Maximum Radiated Emission Calculator:  
Common-mode EMI Algorithm**

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## Abstract

The Maximum Radiated Electromagnetic Emissions Calculator (MREMC) is a software tool that allows the user to calculate the maximum possible radiated emissions that could occur due to specific source geometries on a printed circuit board. This report describes the Common-mode (CM) algorithm, which determines the maximum possible radiated emissions that could occur when a signal on a microstrip trace induces common-mode currents on the cables attached to the circuit board. The methods used, calculations made, and implementation details are described.

## 1. Introduction

A very common source of unwanted radiated emissions from electronic devices is the common-mode current induced on attached cables. Energy from signal currents can be coupled to attached cables through electric or magnetic fields. High frequency signals on a circuit board trace can couple energy to the cables attached to the ground plane directly through their electric field. They can also couple energy to the cables through the magnetic field wrapping around the ground plane generated by the signal currents returning through the finite-impedance ground plane. Both mechanisms can induce CM currents on the cables resulting in radiated emissions. The first source mechanism is referred to as *electric-field coupling*, by which, the magnitude of the induced CM current is proportional to the signal voltage, but independent of the signal current. The second source mechanism is referred to as *magnetic-field coupling*, by which the magnitude of the induced CM current is proportional to the signal current, but independent of the signal voltage. The CM EMI calculator was developed to calculate the maximum possible radiated emissions from structures like this due to the two coupling mechanisms. The calculator utilizes models for equivalent noise source calculations described by Su [1] and expressions for the maximum radiated emissions from PCB-cable structures developed by Deng [2] and Su [3]. This report is an extension of the method described above and is intended to provide details of the implementation sufficient to allow others to develop their own version of this calculator.

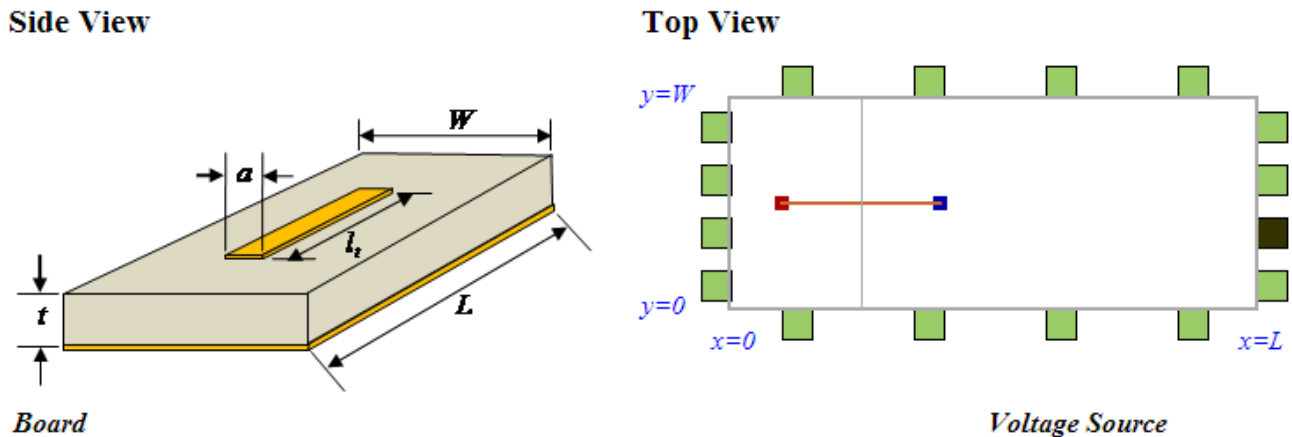


Fig. 1. CM EMI model: (a) side view, (b) top view.

A simple circuit board with a microstrip trace and a ground plane is illustrated in Fig. 1. The circuit board has a length,  $L$ , a width,  $W$ , and a dielectric layer thickness,  $t$ . The signal trace has a width,  $a$ , and a length,  $l_t$ . The positions of the trace and the attached cables are other geometrical parameters required for this calculation. The coordinates of the two end points of the trace are entered into the calculator manually. Cable attachment points (connector positions) are to be chosen from the 16 position options around the perimeter of the circuit board indicated by the green squares in Fig. 1(b).  $R_L$  and  $C_L$  represent the load resistance and capacitance, respectively. The user can choose one of them depending on whether the signal terminates in a CMOS component or a matched load.

The calculator calculates the maximum radiated electric field due to both coupling mechanisms at a distance of 3 meters from the board. It plots the results in  $\text{dB}\mu\text{V}/\text{m}$  from 0 to 500 MHz if the “*Digital Signal*” source type is chosen; or from  $f_0$  to  $f_1$  if the “*Swept Frequency*” source type is chosen. A representative output plot is shown in Fig. 2.  $f_0$  and  $f_1$  are the lower and the upper limits for the frequency sweep respectively.

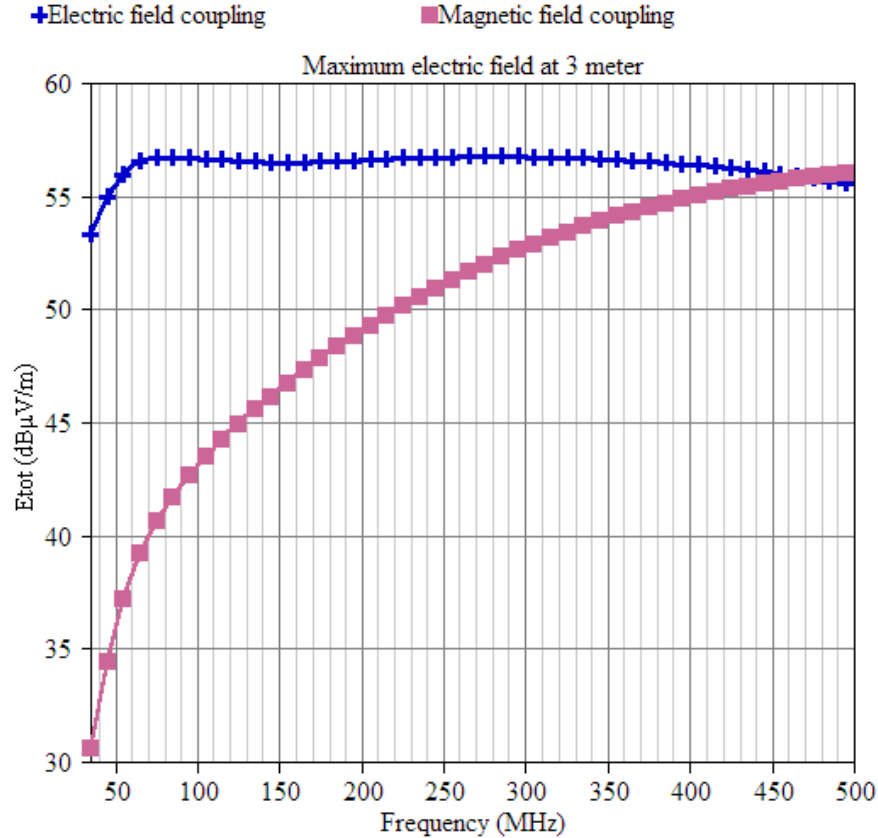


Fig. 2. MREMC output example.

## 2. Description of Algorithm

The algorithm used by the calculator can be broken into two main parts. The first part determines the equivalent CM source based on the source geometry using the *CM Source* algorithm. The second part determines the maximum radiated emissions based on the CM source and the cable-board geometry using the *Radiated Emission Estimation* algorithm. Both parts can be further broken into two subparts: the *Electrical Coupling* algorithm and the *Magnetic Coupling* algorithm.

### 2.1 The CM Source Algorithm

In [1], Su described a method called the Imbalance Difference Method to model the differential-mode (DM) to CM conversion of a signal routed on a trace over a solid ground plane with cables attached to both sides of the ground plane, as shown in Fig. 3(a). The equivalent model is shown in Fig. 3(b), where the trace and the loads are replaced by two CM voltages on the ground plane.  $h_1$ ,  $h_2$ ,  $h_3$  are the imbalance factors, which can be defined for any transmission line geometry and are used to calculate the magnitude of the CM voltages. They can be calculated using the equation,

$$h = \frac{C_{trace}}{C_{trace} + C_{board}} \quad (1)$$

where,  $C_{trace}$  and  $C_{board}$  are the stray capacitances per unit length of the signal trace and ground plane. Note that the imbalance factor  $h$  is always between 0 and 0.5.

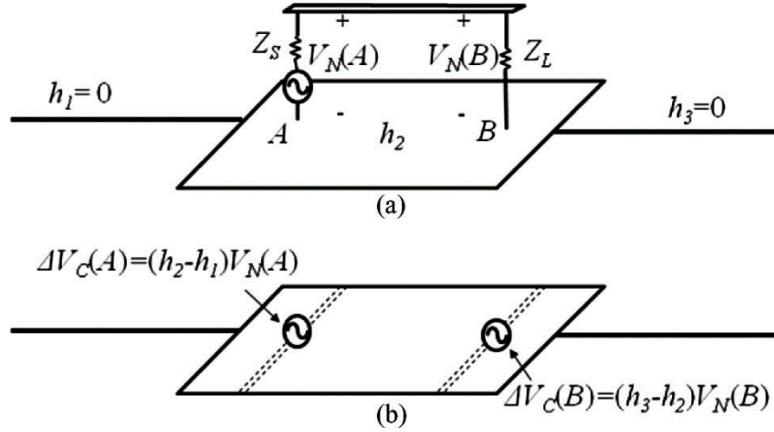


Fig. 3. Imbalance difference model: (a) trace-and-board configuration, (b) equivalent model.

In Fig. 3, there is a change in the imbalance factor  $h$  at both ends of the microstrip. As a result, voltages are generated that drive common-mode currents in the ground plane. These voltages have amplitudes,

$$\Delta V_C(A) = (h_2 - h_1)V_N(A) \quad (2)$$

$$\Delta V_C(B) = (h_3 - h_2)V_N(B) \quad (3)$$

Since  $h_1$  and  $h_3$  are both zero (there is no trace, so  $C_{trace}=0$ ), (2) and (3) can be rewritten as,

$$\Delta V_C(A) = h_2V_N(A) \quad (4)$$

$$\Delta V_C(B) = -h_2V_N(B) \quad (5)$$

If  $V_N(B)$  is the signal on the load end of the circuit,  $V_N(A)$  can be expressed in terms of  $V_N(B)$  as,

$$V_N(A) = V_N(B) + j2\pi f(L_{trace} + L_{return})I_{DM} \quad (6)$$

where  $L_{trace}$  and  $L_{return}$  are the partial inductance of the trace and the board respectively. Combining (4) and (6), we have,

$$\Delta V_C(A) = h_2V_N(B) + j2\pi f(L_{trace} + L_{return})I_{DM} \quad (7)$$

The two CM source amplitudes obtained from (5) and (7) drive the common-mode currents on the structure. Their magnitudes and phases depend on  $V_N(B)$  and  $I_{DM}$  given the imbalance factor  $h_2$  is a constant. As a result, we can further decompose the radiated emissions source into two parts. One part depends on the signal voltage,  $V_N(B)$ , and is the electric-field coupled component. The other part depends on the signal current,  $I_{DM}$ , is the magnetic-field coupled component. Separating the two coupling mechanisms allows users to better understand the cause of the radiated emissions from the circuit board.

### 2.1.1 Electric-Field Coupling

The source components representing the electric-field coupling can be derived by making the circuit in Fig. 3(a) an open circuit as shown in Fig. 4(a), so that the DM current,  $I_{DM}$ , becomes zero. This configuration results when the two CM voltages have the same magnitude and are 180° out of

phase. In this case, the sources drive the attached cables against the board and the induced CM currents flow in opposite directions on cables attached to each side of the board, as shown in Fig. 4(b). The magnitudes of the electric-field component of the CM voltages can be calculated by,

$$V_{CM} = hV_{DM} \quad (8)$$

where  $h$  and  $V_{DM}$  are the same as  $h_2$  and  $V_N(B)$  in Equation (7), (i.e. the imbalance factor of the trace-board geometry and the signal voltage at the load, respectively).

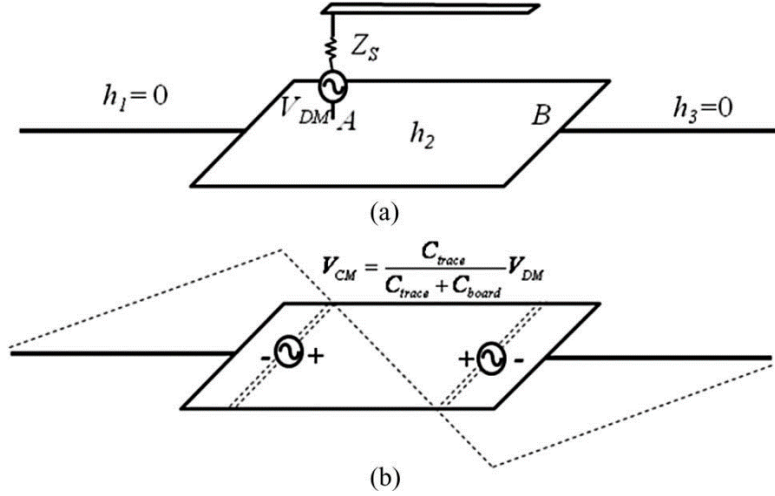


Fig. 4. Imbalance difference model for the open circuit structure.

### 2.1.2 Magnetic-Field Coupling

The sources representing the magnetic-field coupling can be derived by making the circuit in Fig. 3(a) a short circuit as shown in Fig. 5(a). This makes the DM signal voltage,  $V_{DM}$ , zero. The load-end CM voltage is also zero, as shown in Fig. 5(b), leaving only the source end CM voltage with an amplitude that is given by,

$$V_{CM} = h \cdot 2\pi f (L_{trace} + L_{return}) I_{DM} \quad (9)$$

Since  $h$  can also be expressed as,

$$h = \frac{L_{return}}{L_{return} + L_{trace}} \quad (10)$$

Combining (9) and (10), we have,

$$V_{CM} = 2\pi f L_{return} I_{DM} \quad (11)$$

$I_{DM}$  can be found by,

$$I_{DM} = V_{DM} / Z_L \quad (12)$$

where  $Z_L$  is the load impedance. Note that the calculator doesn't allow  $Z_L$  to be zero, because this would cause the signal voltage to also be zero.

The CM voltage obtained from (11) drives one cable relative to another if cables are attached to opposite sides of the board. It drives the cables relative to the board if all cables are attached to the same side of the board. The induced CM currents flow in the same directions on the cables attached to opposite sides of the board, as shown in Fig. 5(b).

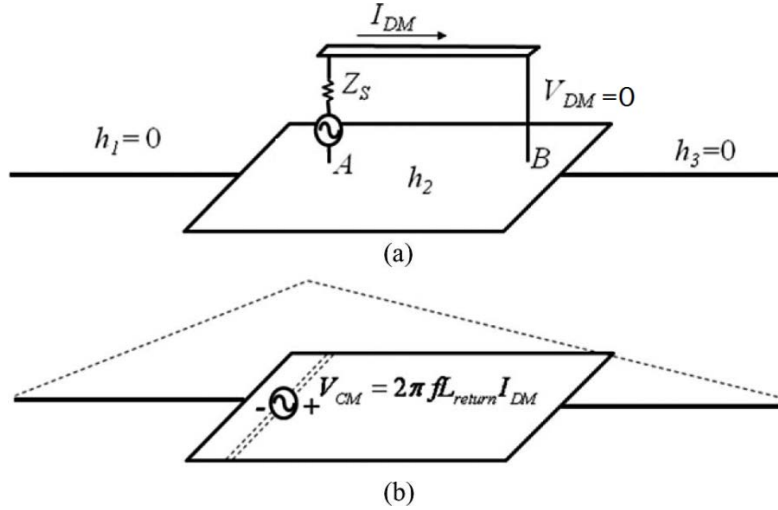


Fig. 5. Imbalance difference model for the shorted trace structure. [1]

### 2.1.3 Calculating the Imbalance Factor

The calculator calculates the imbalance factor,  $h$ , using (10).  $L_{trace}$  is obtained by [4],

$$L_{trace} = \begin{cases} \frac{\mu_0}{2\pi} \ln\left(\frac{8t}{a} + \frac{a}{4t}\right) l_t & a/t \leq 1 \\ \frac{\mu_0}{a/t + 1.393 + 0.667 \ln(a/t + 1.444)} l_t & a/t > 1 \end{cases}, \quad (13)$$

where  $\mu_0$  is the permeability of free space and  $t$  and  $l_t$  are the dielectric thickness and trace length, respectively.  $L_{return}$  is calculated by [5],

$$L_{return} = \frac{\mu_0 t l_t}{\pi W} \frac{1}{\sqrt{1 - 4(1 - 2t/W)(s/W)^2}}, \quad (14)$$

where  $s$  is the offset of the trace from the center of the board and  $W$  is the board width as shown in Fig. 6. The algorithm will calculate the coordinates of the trace center,  $C$ , and obtain the offset,  $s$ , by,

$$s = \frac{|W - 2C_y|}{2}, \quad (15)$$

where  $C_y$  is the  $y$  coordinate of point  $C$ . Note that when the trace is at the corner of the board as shown in Fig. 7, the magnetic field generated by the returning current can wrap around the board's corner instead of the whole width of the board and thus, make  $L_{return}$  larger. To avoid underestimating  $L_{return}$ , the algorithm will replace  $W$  in (14) by  $dist1 + dist2$ , the sum of the distances from the trace center to the two nearest board edges, when the trace is located at the corner of the board as shown in Fig. 7. The offset,  $s$ , is correspondingly replaced by,

$$s = \frac{|dist1 - dist2|}{2}. \quad (16)$$

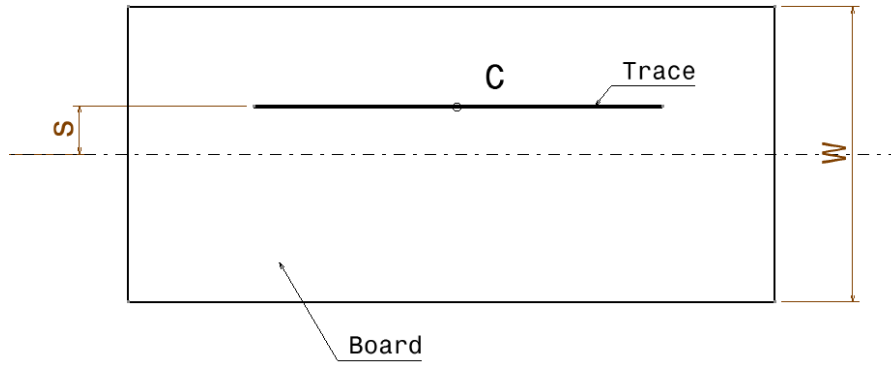


Fig. 6. Trace position relative to the board.

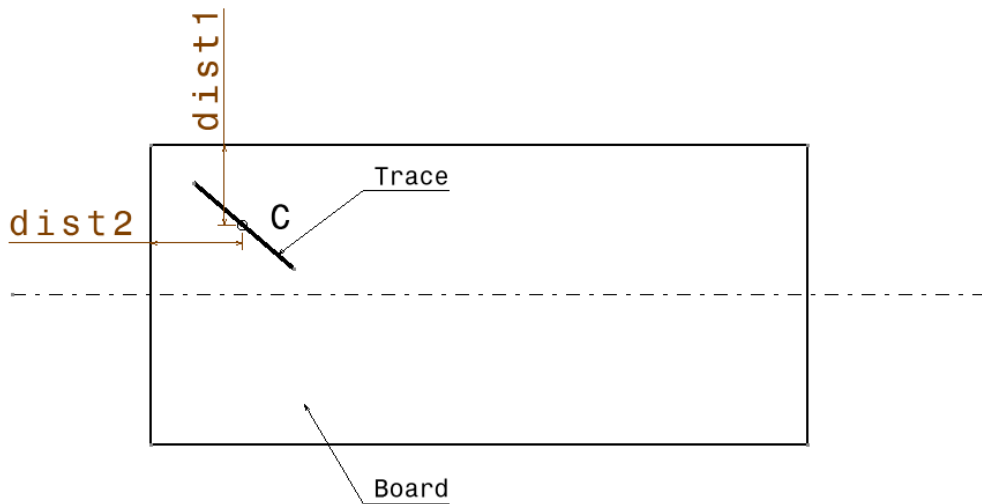


Fig. 7. Trace at the corner of the board.

## 2.2 Radiated Emissions Estimation Algorithm

A detailed description of the radiated emissions estimation algorithm is provided in [6]. The CM emissions calculator supports multiple-cable geometries. It also separates the emissions due to electric-field coupling from the emissions due to magnetic-field coupling. Both components are calculated by the same estimation algorithm in [6] using different effective board lengths for the different coupling mechanisms.

### 2.2.1 Electric-Field Coupling

As shown in Fig. 4(b), the out-of-phase components of the two CM noise sources are responsible for electric-field coupling. These source components drive the attached cables relative to the board. Different board-cable configurations are treated individually by the algorithm to calculate the effective board length. Fig. 8 shows a PCB layout with horizontally and vertically oriented traces. The angle between the trace and the board centerline is beta. If beta is smaller than 45 degrees, the trace is considered horizontally oriented. Otherwise, it is considered to be vertically oriented. As documented in [6], when a cable is driven relative to the board, an estimate of the effective board length is required to calculate the radiated emissions. The effective board length is determined by assigning the trace-board-cable configurations to one of four cases.



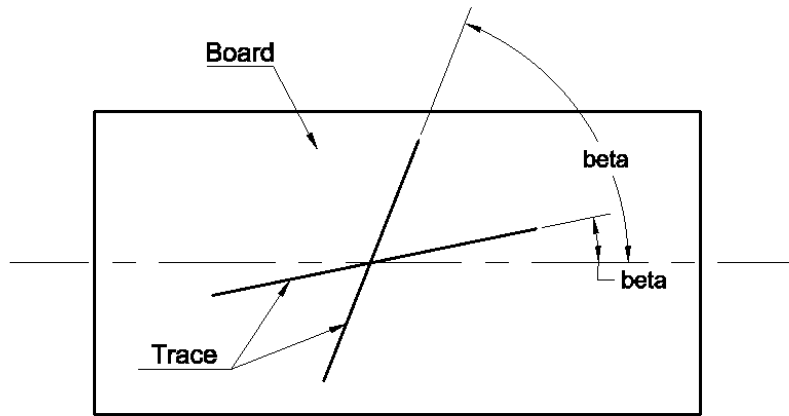


Fig. 8. Trace orientation.

*Case 1: Horizontal trace with cables attached to one side. (Fig. 9)*

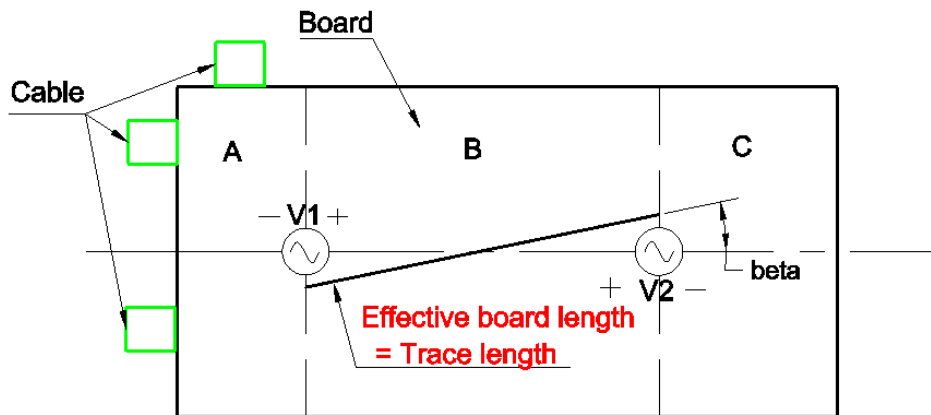


Fig. 9. Electric field coupling: Horizontal trace with cables attached to one side.

This is equivalent to the single-source-single-cable case in [6], except that the effective board length used to calculate the board factor is different. In this case, the CM source,  $V_1$ , is driving board region A and attached cables against board region B as shown in Fig. 9, so the algorithm sets the effective board length equal to the trace length.

Case 2: Horizontal trace with cables attached to both sides. (Fig. 10)

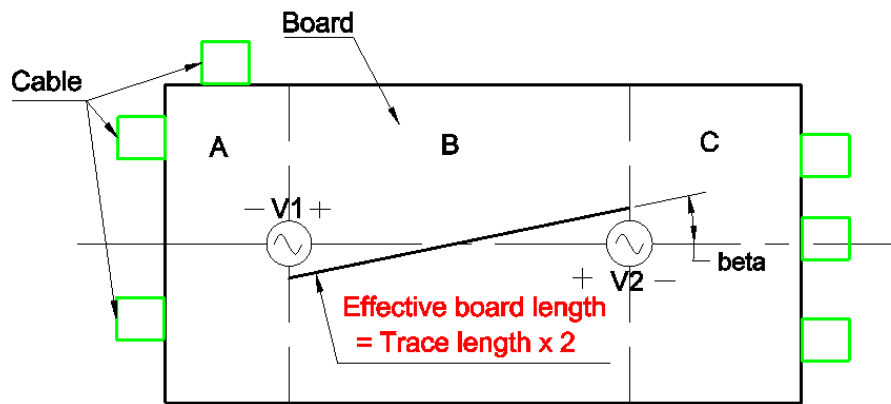


Fig. 10. Electric field coupling: Horizontal trace with cables attached to both sides.

If the cables are attached to opposite ends of the board as shown in Fig. 10, the CM source V1 will drive the cables attached to area A against the board area B and the source V2 will drive the cables attached to the area C against the board area B. The algorithm handles this case by setting the effective board length equal to twice the trace length. While this is not an exact solution, it is a reasonable worst-case approximation for electrically small boards.

Cables attached to board area B are treated as though they were attached to one side of the board. In other words, if there are already cables attached to area A and C of the board, cables attached to B will have no effect on the effective board length.

Case 3: Vertical trace with cables attached to one side. (Fig. 11)

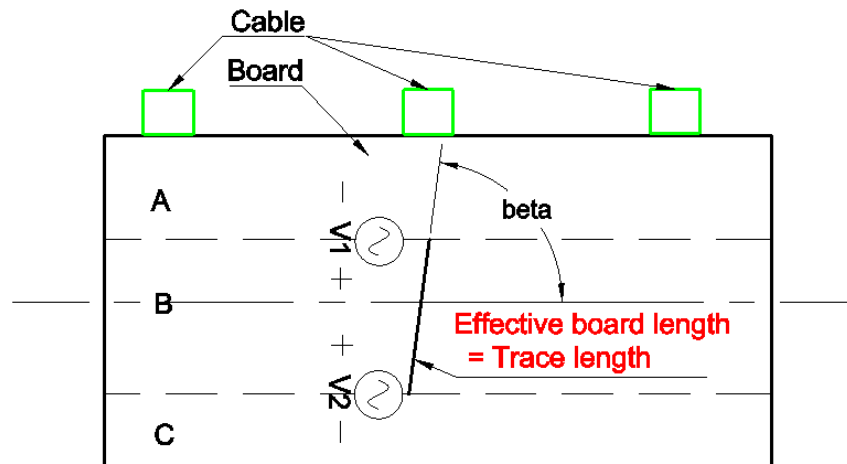


Fig. 11. Electric field coupling: vertical trace with cables attached to one side.

When the angle between the trace and the board center line exceeds 45 degrees, the algorithm considers the trace to be vertically oriented and arranges the board areas A, B and C as shown in Fig. 11. Calculation of the effective board length corresponding to the different cable positions is same as the cases where the trace is horizontally oriented.

In Case 3, all cables are attached to one side of the board (area A or C), so the algorithm uses the trace length for the effective board length.

*Case 4: Vertical trace with cables attached to both sides. (Fig. 12)*

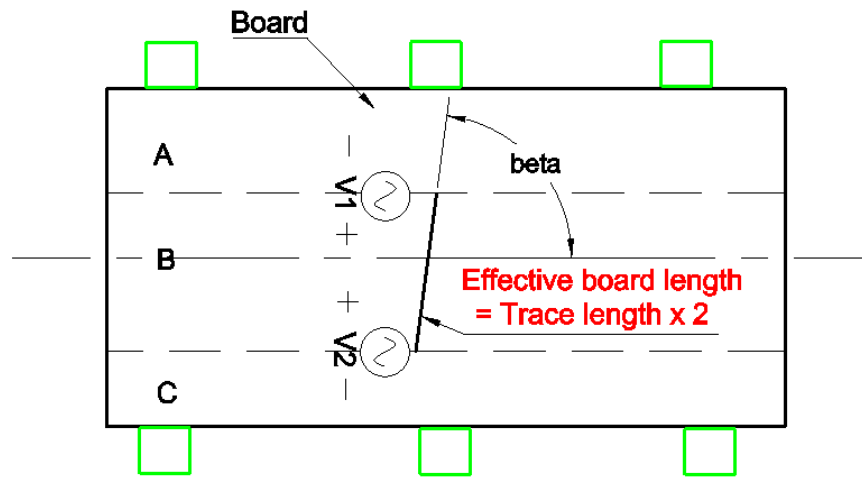


Fig. 12. Electric field coupling: vertical trace with cables attached to one side.

In Case 4, the cables are attached to both sides of the board, so the algorithm makes the effective board length equal to twice the length of the trace.

## 2.2.2 Magnetic-Field Coupling

As shown in Fig. 5 (b), the magnetic-field coupled component is modeled using only one CM noise source. As a result, the CM source will drive the attached cables against the board, if all cables are attached to one side of the board, and will drive some attached cables against others if they are attached to both sides of the board.

*Case 5: Horizontal trace with cables attached to one side. (Fig. 13)*

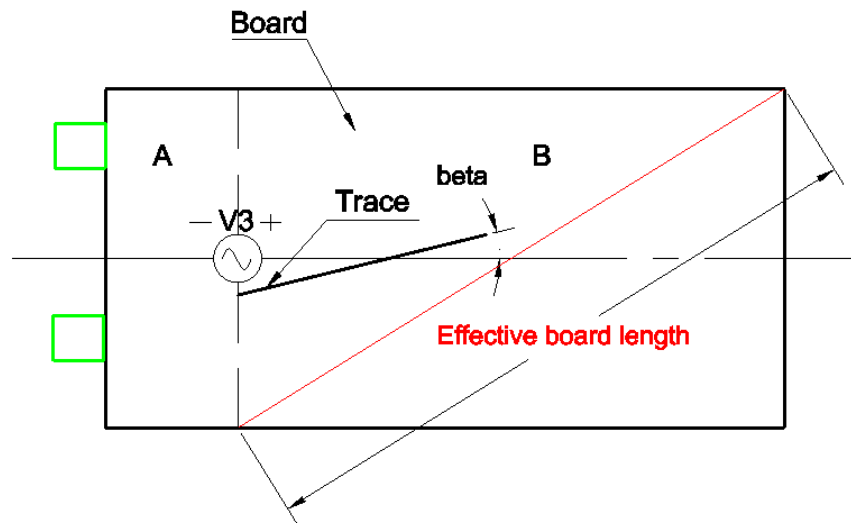


Fig. 13. Magnetic field coupling: Horizontal trace with cables attached to one side.

As shown in Fig. 13, the CM source,  $V_3$ , drives board area A and the cables against board area B. The algorithm uses the diagonal length of the board area B for the effective board length. If the cables

are attached to area B instead of A, the diagonal length of board area A will be used as the effective board length.

*Case 6: Horizontal trace with cables attached to both sides. (Fig. 14)*

In this case, since the CM voltage is driving cables against cables, the algorithm ignores the effective board length and sets the board factor equal to 1.

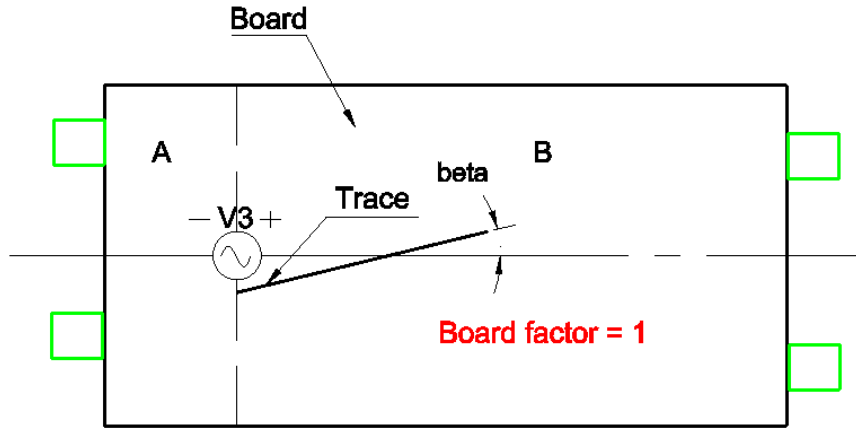


Fig. 14. Magnetic field coupling: Horizontal trace with cables attached to both sides.

*Case 7: Vertical trace with cables attached to one side. (Fig. 15)*

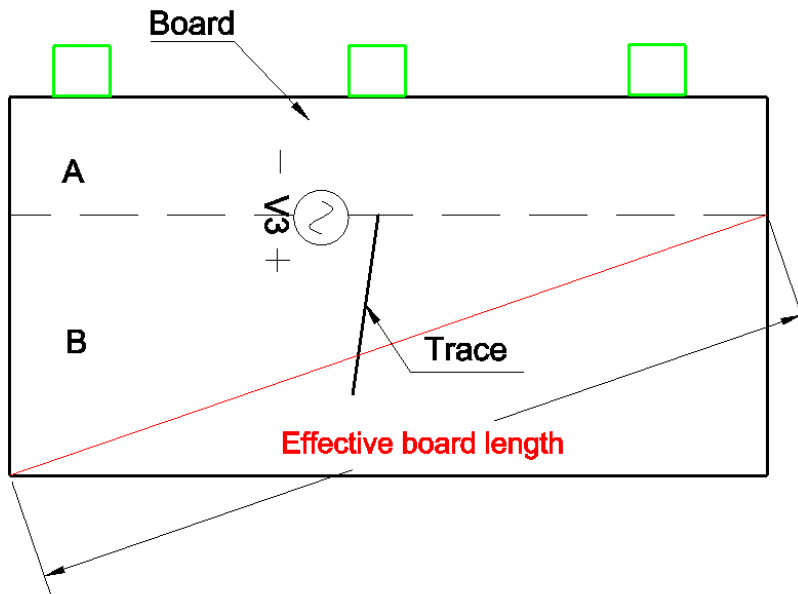


Fig. 15. Magnetic field coupling: Vertical trace with cables attached to one side.

When the trace is vertically oriented, the algorithm arranges the board areas A and B as shown in Fig. 15. Calculation of the effective board length corresponding to the different attached cables positions is same as it is for horizontally oriented traces.

In Case 7, all cables are attached to one side of the board (area A or B), so the algorithm uses the diagonal length of the opposite board area as the effective board length.

Case 8: Vertical trace with cables attached to both sides. (Fig. 16)

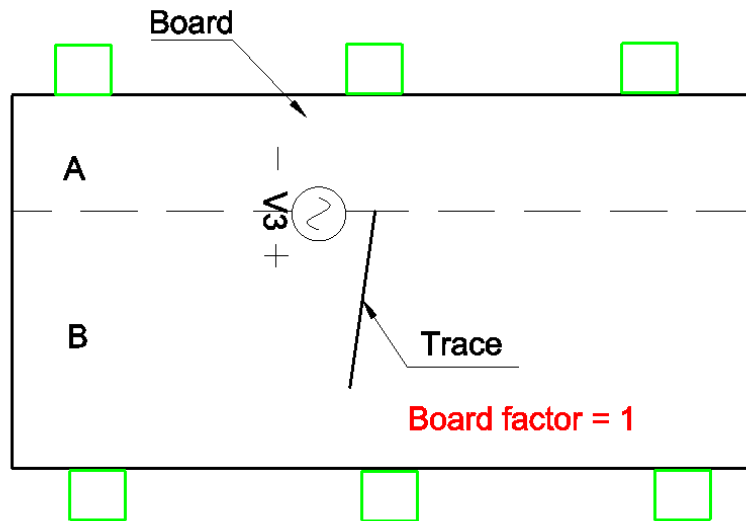


Fig. 16 Magnetic field coupling: Vertical trace with cables attached to both sides.

In Case 8, the cables are attached to both sides of the board, so the algorithm sets the board factor to 1.

### 2.3 Assumptions Made in the Derivation and Implementation of These Algorithms

1. The width of the microstrip trace and the thickness of the dielectric layer are small relative to a wavelength. This ensures the propagation on the trace is quasi-TEM. This assumption was made in order to calculate and apply the imbalance difference method in (10), (13) and (14).
2. The signals are in phase on both ends of the trace. The algorithm does not currently account for any phase shift between the signal at the source end and the signal at the load end.

### 3. Conclusion

This calculator determines the maximum possible radiated emissions due to common-mode currents induced on cables attached to a PCB with a microstrip trace. The current implementation is limited to microstrip traces that are short relative to a wavelength at the highest frequency of the analysis. The algorithm could be extended by using complex values for the differential-mode voltages and equivalent common-mode voltage sources at each end of the trace.

### References

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  - [5] M. Leone, "Design expressions for the trace-to-edge common-mode inductance of a printed circuit board," *IEEE Trans. Electromagn. Compat.*, vol. 43, no. 4, pp. 667–671, 2001.
  - [6] C. Zhu and T. Hubing, "Maximum Radiated Emission Calculator: I/O Coupling Algorithm," *Clemson Vehicular Electronics Laboratory Technical Report, CVEL-13-045*, Aug. 24, 2013.

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## Appendix (Java code)

### h factor calculation in subroutine getData()

```
var we, offset;
we=dist1+dist2;offset=Math.abs(dist1-dist2)/2
lreturn=mu0/pi*t/we/Math.sqrt(1-4*(1-2*t/we)*offset*offset/we/we);
var ltrace;
if (a<=t) ltrace=Math.sqrt(mu0*eps0)*t*60*Math.log(8*t/a+a/4/t);
else ltrace=Math.sqrt(mu0*eps0)*t*120*pi/(a/t+1.393+0.667*Math.log(a/t+1.444));
h=lreturn/(lreturn+ltrace);
```

### Subroutine boardSize(cablepos)

```
function boardSize(cablepos){
    var leff=0;
    var xx1,yy1,xx2,yy2;
    xx1=Math.min(x1,x2);xx2=Math.max(x1,x2);
    yy1=Math.min(y1,y2);yy2=Math.max(y1,y2);
    if (cablepos.length==4){ // voltage-driven type
        if (cablepos[0]==0){ // horizontally arranged
            if ((cablepos[1]==1) && (cablepos[3]==1))
                leff=2*Math.sqrt((xx1-xx2)*(xx1-xx2)+(yy1-yy2)*(yy1-yy2));
            else
                leff=Math.sqrt((xx1-xx2)*(xx1-xx2)+(yy1-yy2)*(yy1-yy2));
        }
        else{ // vertically arranged
            if ((cablepos[1]==1) && (cablepos[3]==1))
                leff=2*Math.sqrt((xx1-xx2)*(xx1-xx2)+(yy1-yy2)*(yy1-yy2));
            else
                leff=Math.sqrt((xx1-xx2)*(xx1-xx2)+(yy1-yy2)*(yy1-yy2));
        }
    }
    else{ // current-driven type
        if (cablepos[0]==0){ // horizontally arranged
            if (cablepos[1]==1)
                leff=Math.sqrt((1-x1)*(1-x1)+w*w);
            else
                leff=Math.sqrt(x1*x1+w*w);
        }
        else{ // vertically arranged
            if (cablepos[1]==1)
                leff=Math.sqrt((w-y1)*(w-y1)+1*1);
            else
                leff=Math.sqrt(y1*y1+1*1);
        }
    }
    return leff;
}
```

### Subroutine calcVDEM()

```
function calcVDEM(){
    // --- Vcm, Icm
    var vcm =new Array();
    for (i=0;i<vdm.length;i++)
        vcm[i]=h*vdm[i];

    // --- determine how many cables are connected to the board
    var cablepos=countCable(0);
```

```

// --- maximum radiation
var emax=0,cablefactor=0,boardfactor=0,lambda=0;
var ymin,ymax;
y=new Array();
pos=new Array(0,0);
var leff=boardSize(cablepos);
for (i=0;i<vcm.length;i++){
    lambda=c0/(x[i]*1e6);
    if (lambda>=4*leff) boardfactor=Math.sin(2*pi*leff/lambda);
    else boardfactor=1;
    if (lambda>=4) cablefactor=Math.sin(2*pi/lambda);
    else cablefactor=1;
    fymax=findFmax(x[i]);
    emax=fymax*emconst*vcm[i]*boardfactor*cablefactor;
    if (emax<1e-15)
        y[i]=0;
    else
        y[i]=sigNumber(20*Math.log(emax*1e6)*Math.LOG10E);
}
}

```

### Subroutine calcCDEM()

```

function calcCDEM(){
    // --- compute Vcm, Icm
    var vcm =new Array();
    for (i=0;i<vdm.length;i++)
        vcm[i]=2*pi*x[i]*1e6*lreturn*idm[i];

    // --- determine how many cables are connected to the board
    var cablepos=countCable(1);
    var cableno=cablepos[1]+cablepos[2];

    // --- maximum radiation
    y=new Array();
    pos=new Array(0,0);
    var leff=boardSize(cablepos);

    var emax=0,lambda=0,boardfactor=0,cablefactor=0;
    var ymin,ymax;
    for (i=0;i<vcm.length;i++){
        lambda=c0/(x[i]*1e6);
        if (lambda>=4*leff) boardfactor=Math.sin(2*pi*leff/lambda);
        else boardfactor=1;
        if (lambda>=4) cablefactor=Math.sin(2*pi/lambda);
        else cablefactor=1;
        fymax=findFmax(x[i]);
        if (cableno==1)
            emax=fymax*emconst*vcm[i]*boardfactor*cablefactor;
        else
            emax=fymax*emconst*vcm[i]*cablefactor;
        if (emax<1e-15)
            y[i]=0;
        else
            y[i]=sigNumber(20*Math.log(emax*1e6)*Math.LOG10E);
    }
}
}

```