2010 Asia-Pacific International Symposium on Electromagnetic Compatibility, April 12 - 16, 2010, Beijing, China

Full-Wave Electromagnetic Modelling from DC to GHz using FEM-SPICE

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Abstract—**A full-wave FEM-SPICE technique is presented for modelling electromagnetic fields from DC to GHz. Previous implementations of this method have not worked well at low frequencies, because the circuit models generated from the finite element formulation had low-frequency stability problems. A modified LU recombination method is here applied to the standard FEM-SPICE formulation in order to eliminate the low-frequency stability problem while allowing a reduction of the number of circuit elements. The equivalent circuit derived from the reformulated equations is suitable for analysis in the time and/or frequency domains by any circuit solver that can model large numbers of linear or non-linear lumped elements. Examples are provided that demonstrate the ability of the new technique to model geometries from DC to several GHz in a single simulation.**

I. Introduction

Combining full-wave electromagnetic (EM) and circuit simulation in a single analysis technique offers many advantages. Several numerical tools suitable for analyzing field-circuit coupled problems have been developed [1-6]. The circuit-oriented finite element method (FEM) based on the finite element simulation of the field domain [3-6] combines the flexible mesh generation and analysis capabilities of FEM with the fast solution capabilities of circuit solvers, such as SPICE. This approach is also well suited for analyzing problems that involve complex distributed EM geometries as well as lumped circuit elements. However, because it is derived from a full-wave FEM formulation, circuit-oriented FEM is hampered by low-frequency instability problems and can not be used to analyze low-frequency problems or geometries driven by a source signal that has a DC component.

Full-wave EM simulation algorithms suffer from low-frequency instability due to the decoupling of the electric and magnetic fields at low frequencies. This problem related to FEM was analyzed in [7]. The matrix formulations can be viewed as a sum of two parts: the first part scales linearly with frequency and therefore is very small at low frequencies; the second part dominates at low frequencies and is singular. The overall matrix is poorly conditioned at low frequencies and the information of the first part can be obscured by numerical error in the second part when the two parts are summed. The LU recombination method proposed in [7] enforces the singularity property of the second part in a manner that ensures that the information contained in the first part won't be neglected when the two parts are added.

In this paper, the low-frequency limitations of the FEM-SPICE method are explored. The LU recombination method is reformulated to modify the FEM matrix and a new equivalent circuit is derived. This approach significantly reduces the number of components in the SPICE simulations and produces circuits that accurately model full-wave problem geometries from DC to the highest valid frequencies of the original FEM formulation.

II. FEM-SPICE Formulation

The frequency domain vector wave equation in terms of the electric field $\boldsymbol{E}$ for a Debye material is give by [6]

$$\nabla\times\frac{1}{j\omega\mu}\nabla\times\boldsymbol{E}+\sigma_0\,\boldsymbol{E}+j\omega\varepsilon_0[\varepsilon_\infty+\chi(\omega)]\boldsymbol{E}=-\boldsymbol{J}_s, \qquad (1)$$

where $\boldsymbol{J}_s$ is the current density of an impressed source, ω the angular frequency, μ the permeability of the medium, and $\chi(\omega)$ the electric susceptibility. For a first order Debye medium, $\chi(\omega)$ is given by

$$\chi(\omega)=\frac{\varepsilon_S-\varepsilon_\infty}{1+j\omega\tau_0}, \qquad (2)$$

where ε_∞ is the permittivity as $\omega\to\infty$ and σ_0 is the conductivity as $\omega\to 0$.

The computational domain is discretized into arbitrarily shaped finite elements and the electric field vector $\boldsymbol{E}$ is locally approximated by the Whitney 1-form [9],

$$E(\mathbf{r})\cong\sum_{k=1}^{n}\mathbf{w}_k(\mathbf{r})\ e_k, \qquad (3)$$

where $\mathbf{r}$ is the position vector, n is the number of element edges in a given element, $\mathbf{w}_k(\mathbf{r})$ is the vector trial function associated with the k-th edge, and e_k is the circulation of the electric field along the k-th edge defined as,

$$e_k = \int_{\ell_k} \boldsymbol{E} \cdot \hat{\mathbf{t}}_k \, d\ell \quad (4)$$
,

with $\hat{\mathbf{t}}_k$ the k-th edge tangent unit vector and ℓ_k the k-th edge length.

By using the FE approximation (3), and applying a Galerkin method, equation (1) is discretized into a matrix form [6],

$$\mathbf{Y}\,\mathbf{e} = \mathbf{I_S}, \quad (5)$$

where $\mathbf{e}=[e_1, e_2,.....e_N]^T$ is the vector containing the electric field circulations, $\mathbf{I_S}$ is the global source current vector, $\mathbf{Y}$ is a non-singular sparse symmetric matrix obtained by the assembling process, and N the number of edges. The global admittance matrix can be expressed as sum of four contributions:

$$\mathbf{Y} = \mathbf{Y_L} + \mathbf{Y_R} + \mathbf{Y_C} + \mathbf{Y_{RC}} \quad , \quad (6)$$

where the matrices $\mathbf{Y_L}$, $\mathbf{Y_R}$, $\mathbf{Y_C}$ and $\mathbf{Y_{RC}}$ are obtained by assembling the respective elemental matrices whose coefficients are given by

$$Y^e_{L_{ij}} = \int_V \frac{1}{(j\omega\mu)} (\nabla \times \mathbf{w}_i \cdot \nabla \times \mathbf{w}_j)\, dV \quad (7a)$$

$$Y^e_{R_{ij}} = \int_V \sigma_0 \nabla \times \mathbf{w}_i \cdot \nabla \times \mathbf{w}_j \, dV \quad (7b)$$

$$Y^e_{C_{ij}} = \int_V j\omega\varepsilon_0\varepsilon_\infty \nabla \times \mathbf{w}_i \cdot \nabla \times \mathbf{w}_j \, dV \quad (7c)$$

$$Y^e_{RC_{ij}} = \int_V j\omega\varepsilon_0\chi(\omega) \nabla \times \mathbf{w}_i \cdot \nabla \times \mathbf{w}_j \, dV. \quad (7d)$$

The elements of $\mathbf{Y_L}$, $\mathbf{Y_R}$, $\mathbf{Y_C}$ behave like inductors, resistors, and capacitors, respectively, while the elements of $\mathbf{Y_{RC}}$ behave like RC circuits. Note that dielectric losses are modeled by a resistive term, while Debye dispersion is modeled by an RC branch. The equivalent circuit of Y_{ij} is then given by the parallel connection four branches as shown in Fig. 1 [6].

The system of equations (5) is similar in structure to multiport network equations based on an admittance matrix representation. Each edge of the FE mesh corresponds to a port in the circuit network and the FEM matrix equation (5) can be translated into an equivalent circuit model as shown in Fig. 2. A detailed description of the SPICE circuit representation can be found in [6].

The low-frequency problem inherent in full-wave finite element methods has been addressed in [7]. On one hand, the elements in matrices $\mathbf{Y_C}$ and $\mathbf{Y_{RC}}$ approach zero at arbitrarily low frequencies as seen from (7c)-(7d), and their contribution tends to be lost in numerical errors during the summation in (6). On the other hand, the matrix $\mathbf{Y_L}$, was shown to be singular when using the popular lowest-order curl-conforming basis functions. It has M sets of linearly dependent rows, where M is equal to the total number of internal nodes in the finite element mesh [10]. The condition number of $\mathbf{Y}$ gets large at low frequencies where $\mathbf{Y_L}$, dominates. Therefore, the loss of information in $\mathbf{Y_C}$ and $\mathbf{Y_{RC}}$, although very small, leads to large errors in the solution at low frequencies.

The FEM-SPICE model has a similar low frequency problem because the circuit model is directly derived from the FEM matrix formulation (5). Low frequency instability problems in the SPICE model are the result of the inductors derived from the $\mathbf{Y_L}$ matrix. Moreover, when implementing the equivalent circuit in SPICE, a small resistor must be introduced in series with the inductor since no closed inductor loops are allowed. These artificial resistors result in an additional numerical error that is insignificant as long as the resistance is smaller than the inductor impedance, but it has a significant impact at low frequency.

These problems related to the singular nature of $\mathbf{Y_L}$ can be overcome by the LU recombination method described in the following section.

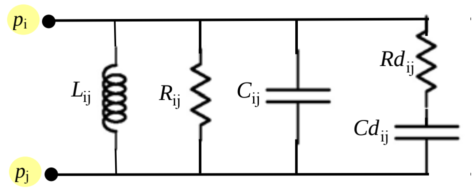

Fig. 1. Equivalent circuit of the mutual admittance Y_{ij} between i-th and j-th ports.

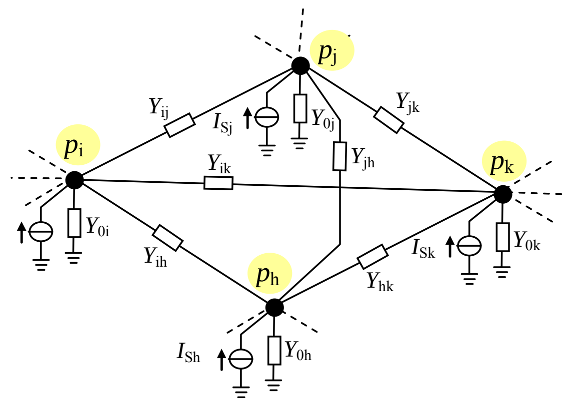

Fig. 2. Norton equivalent circuits corresponding to a portion of the FEM system involving edges i, j, k and h.

III. LU FEM-SPICE FORMULATION

A new approach based on the LU recombination method is proposed to generate a FEM-SPICE model that works at both high and low frequencies. The approach uses the LU recombination method to determine which rows in the matrix $\mathbf{Y_L}$, are linearly dependent. Then for each group of dependent rows, the approach creates a new row that removes the numerical errors. Since the new row is a linear combination of the dependent rows, it can replace any row within the group. The proposed approach also generates a combination matrix $\mathbf{C}$ that records all the dependent rows and how they are linearly related. This matrix is used to incorporate lumped elements and obtain the final solution.

In order to show the LU recombination method using a simple notation, the position $\mathbf{Y}_{\mathrm{HF}} = \mathbf{Y}_{\mathrm{R}} + \mathbf{Y}_{\mathrm{C}} + \mathbf{Y}_{\mathrm{RC}}$ is adopted so that (6) appears in compact form as:

$$\mathbf{Y} = \mathbf{Y}_{\mathrm{L}} + \mathbf{Y}_{\mathrm{HF}}. \tag{8}$$

The detailed process is described using the following example. Let $\mathbf{Y}_{\mathrm{L}i}$ and $\mathbf{Y}_{\mathrm{HF}i}$ (i=1,2,…,N) denote the i-*th* rows of the $\mathbf{Y}_{\mathrm{L}}$ and $\mathbf{Y}_{\mathrm{HF}}$ matrices, respectively. Assume rows *i1*, *i2*, and *i3* are linearly dependent such that,

$$\mathbf{Y}_{\mathrm{L}i1} + \mathbf{Y}_{\mathrm{L}i2} + \mathbf{Y}_{\mathrm{L}i3} = 0. \tag{9}$$

The LU recombination method described in [7] is used to locate these groups of dependent rows. It replaces any of the *i1*, *i2*, and *i3* rows with the linear combination $\mathbf{Y}_0$

$$\begin{aligned}\mathbf{Y}_0 &= \mathbf{Y}_{\mathrm{L}0} + \mathbf{Y}_{\mathrm{HF}} \\ &= (\mathbf{Y}_{\mathrm{L}i1} + \mathbf{Y}_{\mathrm{L}i2} + \mathbf{Y}_{\mathrm{L}i3}) + (\mathbf{Y}_{\mathrm{HF}i1} + \mathbf{Y}_{\mathrm{HF}i2} + \mathbf{Y}_{\mathrm{HF}i3}).\end{aligned} \tag{10}$$

This new row is called a zero-edge row because ideally $\mathbf{Y}_{\mathrm{L}0}$ is zero. A simple way to generate the combination matrix $\mathbf{C}$ is to initialize it as an $N\times N$ identity matrix, with N the total number of edges. Then, for any linearly dependent line *i1* that is intended to be replaced by the zero-edge (10), some matrix coefficients are modified as:

$$C_{i1\ i2} = 1 \qquad\qquad C_{i1\ i3} = 1 \tag{11}$$

The column vector is also replaced to maintain the symmetry, and the following new admittance matrix is obtained:

$$\mathbf{Y}^{new} = \mathbf{C}\,\mathbf{Y}\,\mathbf{C}^{\mathrm{T}} = \mathbf{C}\,\{\mathbf{Y}_{\mathrm{L}} + \mathbf{Y}_{\mathrm{HF}}\}\,\mathbf{C}^{\mathrm{T}}. \tag{12}$$

In practice, the matrix multiplication is performed only on the matrix $\mathbf{Y}_{\mathrm{HF}}$; the *i1* row and column of the matrix $\mathbf{Y}_{\mathrm{L}}$ are set to zero directly. The zero-edge elements have no inductive admittance and therefore circuit models created from the modified admittance matrix $\mathbf{Y}^{new}$ do not have inductors that correspond to these elements or their associated artificial resistors. This effectively removes both the inductances and additional resistances that will affect the low-frequency solution. For each group of dependent rows in $\mathbf{Y}_{\mathrm{L}}$, one row in $\mathbf{Y}$ is replaced by the zero-edge, and all necessary replacements are performed using the matrix multiplication shown in (12). Unlike the original LU recombination method, this new approach does not recover the original finite element matrix, but instead creates a new one.

The new FEM formulation can be expressed in terms of the original formulation as,

$$\mathbf{Y}^{new}\,\mathbf{e}^{new} = \mathbf{I}_{\mathrm{S}}^{new}, \tag{13}$$

where:

$$\mathbf{e} = \mathbf{C}^{T}\,\mathbf{e}^{new} \tag{14a}$$

$$\mathbf{I}_{\mathrm{S}}^{new} = \mathbf{C}\,\mathbf{I}_{\mathrm{S}}. \tag{14b}$$

The new FEM formulation can be translated into a circuit for SPICE simulation using traditional FEM-SPICE techniques. Circuits created from the new formulation will not suffer from low-frequency instability. Note that the linear combination is also applied to the source vector, so extra source elements may be added to the new circuit model. Since the FEM matrix $\mathbf{Y}$ is changed, the solution is also changed. However, the original solution can be easily derived by (14a).

IV. Validation and Discussion

The first example used to validate the approach described in the previous sections is the circuit board power bus model shown in Fig. 3. The structure is composed of two metal planes and a dielectric substrate. The size of the planes is 20 mm × 10 mm, and they are modeled as perfect electric conductors. The dielectric substrate between the two planes has a thickness of 2 mm and a relative permittivity of 4.4. The board is excited by an ideal current source located 2.5 mm and 7.5 mm from the edges at one corner. There is a 50-ohm source resistor in parallel with the source. The fringing field was neglected and the four side walls of the board were modeled as perfect magnetic conductors.

The input impedance was calculated using different methods and the results are shown in Fig. 4. The results obtained using a cavity model [8] are used as a reference, since the cavity model yields accurate results for this type of structure over the entire frequency range evaluated. The dotted line is the result obtained using a normal FEM-SPICE model. The solid line is the result obtained using FEM-SPICE with the LU recombination corrections described in the previous section.

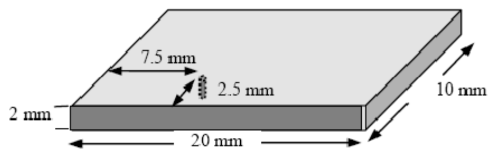

Fig. 1 The geometry of a power bus structure.

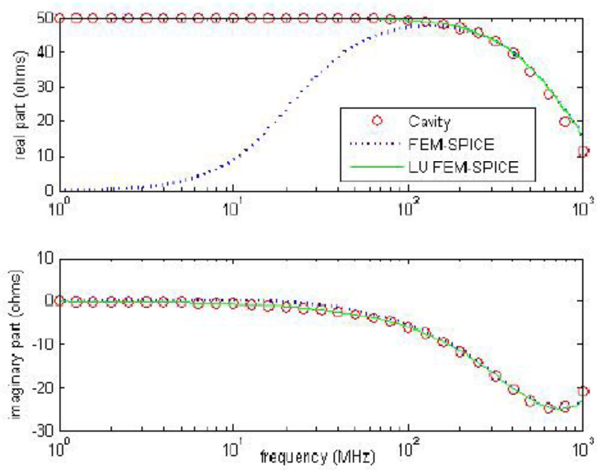

Fig. 4 Power bus structure input impedance.

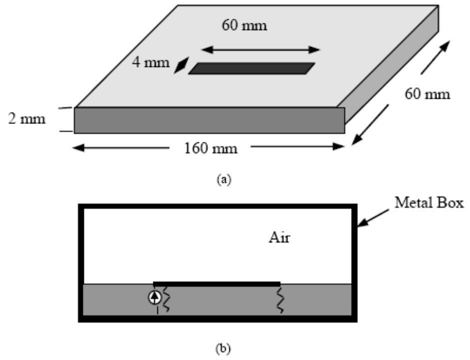

Fig. 5 Metal box example: substrate with trace (a); enclosed by metal box (b).

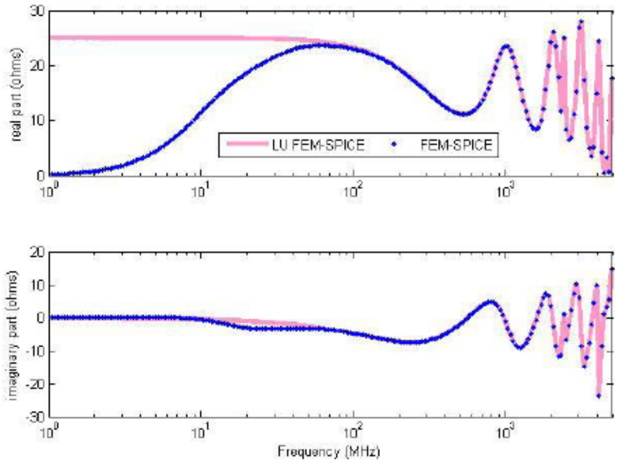

Fig. 6. The input impedance of the trace in a metal box.

The normal FEM-SPICE results behave as if the planes were shorted at low frequencies. After LU recombination, the calculated input impedance matches the cavity model result very well.

The second example is a dielectric substrate with a metal trace on it that is enclosed in a metal box, as shown in Fig. 5. The substrate has dimensions of 160 x 60 x 2 mm and a dielectric constant of 4.4. The trace is 60 mm long and 4 mm wide. The height of the metal box is 8 mm. A 50-ohm source is placed at one end of the trace. A 50-ohm resistor is placed at the other end.

In this configuration, there are two materials in the computational domain, i.e., the dielectric substrate and the air above it. The trace input impedances calculated by FEM-SPICE with and without LU recombination are compared in Fig. 6. Fig. 7 shows the time domain voltage responses at the source and load ends, when the source signal is a trapezoidal wave with a risetime of 50 ps and a period of 5 ns. The result with LU recombination is stable while the result without LU recombination becomes unstable after a couple of cycles. This example demonstrates that material properties affect the value of the FEM matrix elements, but they don't change the linear relationships between the rows of the matrix. The LU recombination method is applicable to configurations containing different materials without additional modification.

More results for the case of a substrate described by a Debye model will be provided in the extended version.

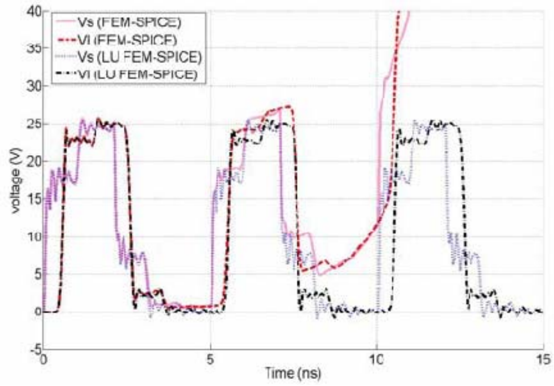

Fig. 7. The time-domain response of the trace in a metal box.

V. Conclusions

FEM-SPICE is a mixed EM field and circuit analysis technique that combines the advantages of FEM and circuit solvers and is well suited for analyzing field-circuit coupled problems. Traditional FEM-SPICE techniques do not work at low frequencies due to numerical instabilities in the FEM formulation and small value resistors that are inserted by SPICE simulators. In this paper the LU recombination method has been applied to FEM-SPICE models in a manner that eliminates the effect of small numerical errors and prevents the unintended insertion of undesired resistors. The new FEM-SPICE formulation works well in frequency domain simulations at arbitrarily low frequencies and can also be used for both DC and transient analyses.

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