Effect of Delay Skew and Transition Time Differences on the Common-Mode Component of Differential Signals

T. Hubing, N. Hubing and C. Guo

October 1, 2001
Executive Summary

Differential signals inherently generate lower radiated emissions than single-ended signals because the fields created by one half of the signal are nearly canceled by the fields created by the other half. However, when the two halves of a differential signal are not perfectly balanced, there is a common-mode component to the signal that can result in radiated EMI. At high frequencies, a slight amount of imbalance can eliminate any advantage obtained by using a differential signaling scheme as opposed to a single-ended signaling scheme.

This technical brief investigates the effect of two types of differential signal imbalance on trapezoidal waveforms. Delay skew and differences in the rise and fall times of a differential signal create imbalance that results in a common-mode voltage. Closed-form expressions for the common-mode voltage are derived in both the time domain and frequency domain. From these expressions, it is possible to quantify the relative impact of these two sources of imbalance.

The results presented here demonstrate that a small amount of signal skew results in a series of common-mode voltage pulses. For a clock signal with a 50% duty cycle, delay skew creates a common-mode pulse train whose energy is primarily at the odd-harmonic frequencies. Transition time differences in the same clock signal produce a pulse train with energy primarily at the even harmonic frequencies. At higher harmonic frequencies, these pulses can have as much energy as a single-ended waveform with the same peak-to-peak amplitude.
1 Introduction

Differential signaling is widely employed for high-speed electronic communication. Differential data transmission schemes (e.g. Fibre channel, LVDS) take advantage of the greater noise immunity and reduced radiated emissions normally associated with differential signals. Theoretically, a perfectly balanced differential data interface consisting of a differential source, a balanced conductor pair (e.g. twisted wire pair or identical co-planar traces on a printed circuit board) and a balanced receiver, will not generate common-mode currents, which are a significant source of radiated emissions [1]. In practice however, imbalances in the transmitter conductor pair and/or receiver generate small amounts of common-mode (even-mode) currents that tend to be the dominant source of radiated emissions.

There are many possible sources of imbalance in differential signal transmission schemes. The voltages generated by the transmitter may not be exactly equal and opposite relative to conductors that are not part of the intended current path. The two differential signal conductors may not be perfectly balanced (i.e. they may have different impedances relative to conductors that are not part of the intended current path). Also, the receiver may introduce different impedances from each signal conductor to a third conductor. Any of these sources of imbalance may cause some of the signal current to flow on a conductor that is not part of the intended current path.

In this paper, we review two symptoms of imbalance in differential interfaces where two wires or traces are driven with an equal and opposite voltage relative to a third conductor (e.g. a ground plane or shield). The first is delay skew, which results when one half of a differential digital signal does not transition at exactly the same time as the other half. The second symptom is imbalance between the rise and fall times of the two sides of the differential signal sometimes referred to as slew-rate skew [2].

2 Delay Skew

Delay skew can result when one side of a differential digital signal source switches at a slightly different time than the other side. It may also result when one trace or wire in the differential pair is longer than the other causing the two halves of the signal to arrive at different times. As illustrated in Fig. 1, when the two halves of the differential signal do not switch at exactly the same time, there is a short burst of common-mode voltage.

Modeling the two sides of the differential signal as trapezoidal waveforms with amplitude A, transitions time $t_r$ and delay skew $\tau_d$, a simple equation for the common-mode voltage can be derived for the case $\tau_d > t_r$. 
In this case, the common-mode waveform is a series of trapezoids with amplitude $A$, transition time $t_r$ and duration $\tau_d + t_r$. When $\tau_d$ is less than $t_r$, the expression in Equation (1) reduces to

$$v(t) = \begin{cases} \frac{At}{t_r} & 0 < t < t_r \\ \frac{A}{t_r} (-t + (t_r + \tau_d)) & \tau_d < t < t_r + \tau_d \end{cases}$$

(1)

In this case, the common-mode waveform is a series of trapezoidal pulses with amplitude $A \tau_d / t_r$.

$$v(t) = \begin{cases} \frac{At}{t_r} & 0 < t < \tau_d \\ \frac{At \tau_d}{t_r} & \tau_d < t < t_r \\ \frac{A}{t_r} (-t + (t_r + \tau_d)) & t_r < t < t_r + \tau_d \end{cases}$$

(2)

In the time domain, this series of trapezoids can be represented by a convolution of a single trapezoid with two periodic pulse trains. One pulse train consists of a unit pulse at $t=0$ and is...
period with period $T$. The second pulse train consists of a negative unit pulse at $t=T/2$ and is also periodic with period $T$.

In the frequency domain, the two pulse trains effectively sample the Fourier transform of the trapezoidal waveform at odd harmonics of the fundamental frequency. Therefore the trapezoidal waveform’s transform provides an envelope of the common-mode voltage in the frequency domain.

The one-sided Fourier series coefficients corresponding to the waveforms in Equations (1) and (2) are the same and are given by:

$$|v(n)| = \begin{cases} \frac{2A\tau_d}{T} \frac{\sin n\pi f_0\tau_d}{n\pi f_0\tau_d} \frac{\sin n\pi f_0t_r}{n\pi f_0t_r} & n = 1, 3, 5, \ldots \\ 0 & n = 2, 4, 6, \ldots \end{cases}$$

where $f_0 = 1/T$ is the fundamental frequency of the original waveform. Note that when $\tau_d$ and $t_r$ are much smaller than $T$ (which is normally the case), the two $\sin(x)/x$ terms have a value near unity. This suggests that the amplitude of the first several harmonics is independent of frequency and directly proportional to the amount of delay skew, $\tau_d$.

![Fig. 2. The envelope of the amplitude of the harmonics in Equation (2) ($t_r > > \tau_d$).](image)

A plot of the envelope of the amplitude of the harmonics for a waveform where $\tau_d << t_r << T$ is shown in Fig. 2. Note that the amplitude of the harmonics does not decrease with frequency below $f=1/\pi t_r$. The common-mode voltage is a series of short pulses. This common-mode signal has more of its power in the higher harmonic frequencies than the original differential signal. Fig. 3 shows the relative amplitude of the first 100 harmonics of the common-mode waveform resulting from a 1% delay skew ($\tau_d=0.01T$) in a differential signal. This is compared to the harmonics in a single-ended clock signal with the same amplitude (i.e. $V_{pp}=2A$).
Fig. 3. Amplitude of first 100 common-mode harmonics of a clock signal with a 50% duty cycle. (single-ended vs. differential with 1% delay skew)

Fig. 4. Amplitude of first 100 common-mode harmonics of a clock signal with a 50% duty cycle. (single-ended vs. differential with 10% delay skew)
Only the odd harmonics are shown, since the even harmonics have zero amplitude. Note that the differential waveform is almost 30 dB lower at the fundamental frequency. However, the power in the harmonics of the differential signal does not fall off with frequency as fast as the power in the harmonics of the single-ended signal. Around the 51st harmonic, the power in both signals is approximately equal.

Fig. 4 shows a similar comparison between a single-ended signal and a differential signal with 10% delay skew. While the amplitudes of some harmonics are significantly reduced, many others are not affected. Of the first 15 harmonics, only the 1st, 9th and 11th are significantly attenuated. In general, the first harmonic that is not reduced at all can be calculated using the formula,

\[ N = \frac{50}{\%\text{delayskew}} \quad (0 \leq \%\text{delay} \leq 50) \quad (4) \]

where N is an integer representing the Nth harmonic. For example, with 10% delay skew, the amplitude of the 5th harmonic of the differential signal is equal to the amplitude of an equivalent single-ended signal.

Note that using one half of a differential clock output for our single-ended signal would have produced harmonics that were 6 dB lower than those indicated in Figs. 3 and 4. This suggests that pseudo-differential clocks with delay skew can actually generate higher emissions at many frequencies than their single-ended equivalents, which have half the peak-to-peak voltage.

---

**Fig. 5.** Common-mode voltage due to transition time differences.
3 Transition Time Differences

Even when there is no delay skew, a common-mode voltage will occur if the rise-time and fall-time of a differential signal are not equal. This is illustrated in Fig. 5. For a trapezoidal waveform, the equation of the induced common-mode voltage is,

\[
v(t) = \begin{cases} 
  \frac{At}{t_1} & 0 < t < \frac{t_1 - t_2}{2} \\
  \frac{A(t_1 - t_2)}{t_1t_2} \left( \frac{t_1}{2} - t \right) & \frac{t_1 - t_2}{2} < t < \frac{t_1 + t_2}{2} \\
  A \left[ \frac{t}{t_1} - 1 \right] & \frac{t_1 + t_2}{2} < t < t_1 
\end{cases}
\]  

(5)

where \( A \) is the amplitude of the source, \( t_1 \) is the longer transition time and \( t_2 \) is the shorter transition time. The resulting waveform has a double-triangle shape as shown in Fig. 5. This waveform has been observed in measurements of the common-mode voltage generated by a commercial differential clock driver [3]. The amplitude of the common-mode voltage is,

\[
V_{PP} = A \left( \frac{t_1 - t_2}{t_1} \right)
\]

(6)

In the time domain, the double-triangle waveform can be represented as the time-integral of the waveform shown in Fig. 6. This waveform can be represented as a sum of two rectangular pulse functions.

Fig. 6. Decomposition of the time-integral of the double-triangle waveform.
In the frequency domain, the Fourier transform of the double-triangle waveform can be obtained by summing the transforms of the two pulse functions on the right-hand side of Fig. 6 and dividing the result by \( j\omega \) (equivalent to taking the time integral). The double-triangle waveform is periodic with period, \( T/2 \). The one-sided Fourier series coefficients corresponding to the common-mode voltage waveform is therefore given by:

\[
|v(n)| = \begin{cases} 
\frac{2A}{n\pi} \sin \frac{n\pi f_0 t_1}{n\pi f_0 t_2} & n = 2,4,6... \\
0 & n = 1,3,5...
\end{cases}
\]

where \( f_0 = 1/T \) is the fundamental frequency of the original waveform. Note that when \( t_1 = t_2 \) (i.e. the rise time equals the fall time), the common-mode voltage is zero.

Although it is difficult to plot a simple envelope for this function that is valid for all possible values of \( t_1, t_2 \) and \( T \), there are a few useful observations that help us to compare the relative effects of transition-time differences versus delay skew. Perhaps the most significant observation is that delay skew (in a signal with a 50% duty cycle) produces odd-harmonics only. Transition time differences produce only even harmonics.

![Fig. 7. Upper bounds on the common-mode voltage due to transition time differences.](image-url)

When \( nf_0 << 1/t_1 \), (i.e. the lower harmonics assuming the rise and fall times are small relative to the period), we can use the small argument approximation for the \( \sin(x)/x \) function, \( \frac{\sin(x)}{x} \approx 1 - \frac{x^2}{3!} \). Equation (7) then reduces to,
\[ |v(n)| \approx \frac{2}{3} n \pi f_o^2 A \left( t_1^2 - t_2^2 \right). \] (8)

At higher frequencies, we can put an upper bound on (6) by noting that the amplitude of the difference between the two \( \sin(x)/x \) functions can be no higher than the sum of their absolute values and that \( \sin(x) \) is always less than or equal to 1,

\[ |v(n)| \leq \frac{A}{2 \pi^2 n^2 f_o} \left( \frac{t_1 + t_2}{t_1 t_2} \right) \] (9)

Applying the bounds in (8) and (9), an envelope of the common-mode voltage due to a transition time difference is illustrated in Fig. 7.

### 4 Conclusions

Both delay skew and differences between the rise and fall times produce common-mode voltages on differential signal lines. These common-mode voltages create common-mode currents that may be a significant source of radiated EMI. The common-mode voltage resulting from delay skew appears at odd harmonics of the fundamental and has an amplitude directly proportional to the amount of delay. The common-mode voltage resulting from transition time differences appears at even harmonics.

The amplitude of the common-mode voltage at any harmonic frequency is never more than the sum of the voltages at that frequency due to each half of the differential signal. At low harmonic frequencies, the contributions to the common-mode voltage from each half of the differential signal tend to cancel each other. However, with a little skew introduced, the contributions to the common-mode voltage from each half add at the higher harmonic frequencies. This can result in radiated emissions from a differential signal that are actually higher than the emissions due to half of the clock output operating single ended.

### References

